

TD 02

Exercise 1

Let $A, B, C \in P(E)$, proving the following :

1. $A \setminus (B \cap C) = (A \setminus B) \cup (A \setminus C)$.
2. $A \subset B \Leftrightarrow A \cup B = B$
3. $A \cup B = A \cap C \Leftrightarrow B \subset A \subset C$

Exercise 2

Given A, B and C three parts of a set E ,

1. Show that :
 - (a) $(A \cap B) \cup \complement_E B = A \cup \complement_E B$.
 - (b) $(A \setminus B) \setminus C = A \setminus (B \cup C)$.
2. Simplify :
 - (a) $\overline{(A \cup B) \cap (C \cup \overline{A})}$.
 - (b) $\overline{(A \cap B) \cup (C \cap \overline{A})}$.

Exercise 3

Let E, F two sets, $f : E \rightarrow F$ is an application, proving the following :

1. $\forall A, B \in P(E), f(A \cap B) \subset f(A) \cap f(B)$.
2. $\forall A, B \in P(E), f(A \cup B) = f(A) \cup f(B)$.
3. $\forall A \in P(F), f^{-1}(F \setminus A) = E \setminus f^{-1}(A)$.

Exercise 4

1. Describe the direct image of $\mathbb{R}$ by the exponential function
2. Find $f([0, 1[), f(\mathbb{R}), f([-1, 2[), f^{-1}(\{3\})$, par la fonction $f : x \mapsto x^2$, defined on $\mathbb{R}$.

Exercise 5

Let $f : [1; +\infty[\rightarrow [0; +\infty[$ is a function such that $f(x) = x^2 - 1$ bijective ?

Exercise 6

Let E, F, G three sets, $f : E \rightarrow F, g : F \rightarrow G$ and $h : G \rightarrow E$

Proving that is $h \circ g \circ f$ is injective and if $g \circ f \circ h$ and $f \circ h \circ g$ are surjectives. Then, f, g and h are bijectives.