

Tutorial 02 : Analysis vectorial

Exercise 01 :

For the following champs, determine the gradient, divergence, and curl

Where applicable.

$$f(x, y, z) = x^3 - 2x^2yz - y^3z + 5 \quad , \quad F(x, y, z) = 2x\vec{i} + 3yz\vec{j} - 4xz^2\vec{k}$$

$$G(x, y, z) = yx\cos(x)\vec{i} + (-2x^2 + z)\vec{j} - (5 - z)\vec{k}$$

Exercise 02 :

1) Consider the scalar field $f(x, y, z) = 3x^2y - y^3z$.

Calculate : $\text{Grad} f$, $\text{div}(\text{grad} f)$ at the point $(1, -2, 1)$.

2) Let the vector field defined by : $F(x, y, z) = x^2z\vec{i} - 2y^3z^2\vec{j} + xzy^2\vec{k}$.

Calculate : $\text{Div} F$, $\text{Grad}(\text{Div} F)$ at point $(1, -1, 1)$.

3) Let the vector field given by : $G(x, y, z) = xz^3\vec{i} - 2x^2yz\vec{j} + 2yz^4\vec{k}$.

Calculate : $\text{Rot} F$, $\text{Div}(\text{Rot} F)$, and $\text{Grad}(\text{Div}(\text{Rot} F))$ at point $(1, 1, 0)$.

Exercise 03 :

Let f be a scalar field, and F, G two vector fields. Show that

$$\text{Div}(\text{Rot} F) = 0 \quad , \quad \text{Rot}(\text{Grad} f) = 0 \quad , \quad \text{Div}(F \times G) = G \cdot \text{Rot}(F) - F \cdot \text{Rot}(G)$$

$$\text{Rot}(\text{Rot} G) = \text{Grad}(\text{Div} G) - \Delta G$$

Exercise 04 :

Let $F(x, y, z) = 3x^2z^2\vec{i} + 2x^2\vec{j} - 2x^3z\vec{k}$ be a vector field.

1) Show that the vector field F is irrotational. ($\text{Rot} F = 0$).

2) Deduce the existence of a scalar function f such that :

$$F = \text{Grad} f.$$

3) find a potential from which it derives

$$G(x, y, z) = (3x^2 + 3y - 1)\vec{i} + (3x + z^2)\vec{j} + (2yz + 1)\vec{k}$$