

Course 1 : **Scalar & vector**

In Mathematics and physics a quantities can be classified by two main categories, scalars and vectors.

Scalar : Is quantitiy having just magnitude,can be represented by by a numerical value.

Exemple : Temperature, mass, volume, time, density, length, ...

Vector : Is quntity having magnitude and Direction.

Example : Displacement, velocity, acceleration, force, ...

A vector can be represented by an arrow.

$\vec{v}$ (letter with arrow) $\vec{v}$ is a vector and $|\vec{v}|$ is its modulus.

If the modulus of vector $\vec{v}$ is equal to 1, we say that $\vec{v}$ is a unit vector.

In the coordinates system : If $\vec{v} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ Then $|\vec{v}| = \sqrt{x^2 + y^2 + z^2}$.

Scalar and Vector product

Scalar product : Let $\vec{u}$ and $\vec{v}$ two vectors.

The scalar product between two vectors $\vec{u}$ and $\vec{v}$ is denoted as $\vec{u} \cdot \vec{v}$.

And $\vec{u} \cdot \vec{v} = |\vec{u}||\vec{v}|\cos(\theta)$, with θ is the angle between the vecors $\vec{u}$ and $\vec{v}$.

In the coordinates system :

$$\text{If } \vec{u} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \text{ and } \vec{v} = \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} \text{ Then } \vec{u} \cdot \vec{v} = xx' + yy' + zz'.$$

Properties : $\vec{u}$, $\vec{v}$ and $\vec{w}$ three vectors. We have :

$$\vec{u} \cdot \vec{v} = \vec{v} \cdot \vec{u}$$

$$\vec{u} \cdot (\vec{v} + \vec{w}) = \vec{u} \cdot \vec{v} + \vec{u} \cdot \vec{w}$$

$$\alpha(\vec{u} \cdot \vec{v}) = (\alpha\vec{u}) \cdot \vec{v} = \vec{u} \cdot (\alpha\vec{v})$$

$$\text{If } \vec{u} \perp \vec{v} \text{ Then } \vec{u} \cdot \vec{v} = 0$$

Vector product

The vector product between two vectors $\vec{u}$ and $\vec{v}$ is denoted as $\vec{u} \times \vec{v}$ (Or $\vec{u} \wedge \vec{v}$).

$\vec{u} \times \vec{v} = |\vec{u}||\vec{v}|\sin(\theta) \vec{n}$, with θ is the angle between the vecors $\vec{u}$ and $\vec{v}$, and $\vec{n}$ is the unit

Vector perpendicular to both the vectors $\vec{u}, \vec{v}$.

In the coordinates system : $(O, \vec{i}, \vec{j}, \vec{k})$ orthonormal basis.

$$\text{If } \vec{u} = x\vec{i} + y\vec{j} + z\vec{k}, \text{ and } \vec{v} = x'\vec{i} + y'\vec{j} + z'\vec{k} \text{ Then } \vec{u} \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x & y & z \\ x' & y' & z' \end{vmatrix}$$

Properties : $\vec{u}, \vec{v}$ and $\vec{w}$ three vectors. We have :

$$|\vec{u} \times \vec{v}| = |\vec{u}||\vec{v}|\sin(\theta)$$

$$\vec{u} \times \vec{v} = -\vec{v} \times \vec{u}$$

$$\vec{u} \times (\vec{v} + \vec{w}) = \vec{u} \times \vec{v} + \vec{u} \times \vec{w}$$

$$\alpha(\vec{u} \times \vec{v}) = (\alpha\vec{u}) \times \vec{v} = \vec{u} \times (\alpha\vec{v})$$

$$\text{If } \vec{u} // \vec{v} \text{ Then } \vec{u} \times \vec{v} = 0$$

$$\vec{u} \times \vec{v} \perp (\vec{u}, \vec{v})$$

The vector product of the two vectors represents the area of the parallelogram formed with these 2 vectors.

Triple product

Scalar triple product (Mixed product) Given $\vec{u}, \vec{v}$ and $\vec{w}$ three vectors in space with an orthonormal Basis $(O, \vec{i}, \vec{j}, \vec{k})$.

$$\text{Mixed product is defined by : } \vec{u} \cdot (\vec{v} \times \vec{w}) = \begin{vmatrix} x & y & z \\ x' & y' & z' \\ x'' & y'' & z'' \end{vmatrix}$$

$$\text{With : } \vec{u} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \vec{v} = \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix}, \text{ and } \vec{w} = \begin{pmatrix} x'' \\ y'' \\ z'' \end{pmatrix}.$$

Properties :

$$\vec{u} \cdot (\vec{v} \times \vec{w}) = (\vec{v} \times \vec{w}) \cdot \vec{u}$$

$$\vec{u} \cdot (\vec{v} \times \vec{w}) = \vec{v} \cdot (\vec{w} \times \vec{u}) = \vec{w} \cdot (\vec{u} \times \vec{v})$$

$|\vec{u} \cdot (\vec{v} \times \vec{w})|$ represents the volume of the parallelepiped formed with these three vectors.

If $\vec{u} \cdot (\vec{v} \times \vec{w}) = 0$, then a vectors $\vec{u}, \vec{v}$ and $\vec{w}$ are said to be coplanar.

Vector triple product

Vector triple product of three vectors $\vec{u}, \vec{v}$ and $\vec{w}$ is denoted by $\vec{u} \times (\vec{v} \times \vec{w})$.

Properties :

$$\vec{u} \times (\vec{v} \times \vec{w}) \neq (\vec{u} \times \vec{v}) \times \vec{w}$$

$$\vec{u} \times (\vec{v} \times \vec{w}) = (\vec{u} \cdot \vec{w})\vec{v} - (\vec{u} \cdot \vec{v})\vec{w}$$

