

TD 01

Exercise 1

Consider the following four statements :

- (a) $\exists x \in \mathbb{R}, \forall y \in \mathbb{R}, x + y > 0$.
- (b) $\forall x \in \mathbb{R}, \exists y \in \mathbb{R}, x + y > 0$.
- (c) $\forall x \in \mathbb{R}, \forall y \in \mathbb{R}, x + y > 0$.
- (d) $\exists x \in \mathbb{R}, \forall y \in \mathbb{R}, y^2 > x$.

- Are the statements a, b, c, d true or false? Provide their negations.
- Let P, Q, and R be three statements. Verify by creating a truth table :
 - (a) $P \wedge (Q \vee R) \Leftrightarrow (P \wedge Q) \vee (P \wedge R)$,
 - (b) $\overline{(P \Rightarrow Q)} \Leftrightarrow P \wedge \overline{Q}$.

Exercise 2

Let f function of $\mathbb{R}$ in $\mathbb{R}$. Translate the following expressions into quantifier terms :

- f is bounded above.
- f is bounded.
- f is even.
- f never equals zero.
- f is periodic.
- f is increasing.
- f is not the zero function.
- f attains all values in $\mathbb{N}$.

Exercise 3

Let $f : \mathbb{R} \rightarrow \mathbb{R}$. What is the difference in meaning between the two proposed statements?

- $\forall x \in \mathbb{R}, \exists y \in \mathbb{R}, y = f(x)$ and $\exists y \in \mathbb{R}, \forall x \in \mathbb{R}, y = f(x)$.
- $\forall y \in \mathbb{R}, \exists x \in \mathbb{R}, y = f(x)$ and $\exists x \in \mathbb{R}, \forall y \in \mathbb{R}, y = f(x)$.
- $\forall x \in \mathbb{R}, \exists M \in \mathbb{R}, f(x) \leq M$ and $\exists M \in \mathbb{R}, \forall x \in \mathbb{R}, f(x) \leq M$.

Exercise 4

Show by recurrence that :

- $\forall n \in \mathbb{N}^* : 1^3 + 2^3 + \dots + n^3 = \frac{n^2(n+1)^2}{4}$
- $\forall n \in \mathbb{N}, 4^n + 6n - 1$ is a multiple of 9.

Exercise 5

By the absurd show that :

$$\forall n \in \mathbb{N}, n^2 \text{ even} \Rightarrow n \text{ is even.}$$

Exercise 6

By contrapositive, show that

- If $(n^2 - 1)$ is not divisible by 8 then n is even.
- $\forall \varepsilon > 0, |x| \leq \varepsilon \Rightarrow x = 0$.