Ex.4 :

- *Degrees of freedom (mobility) and cyclomatic number*

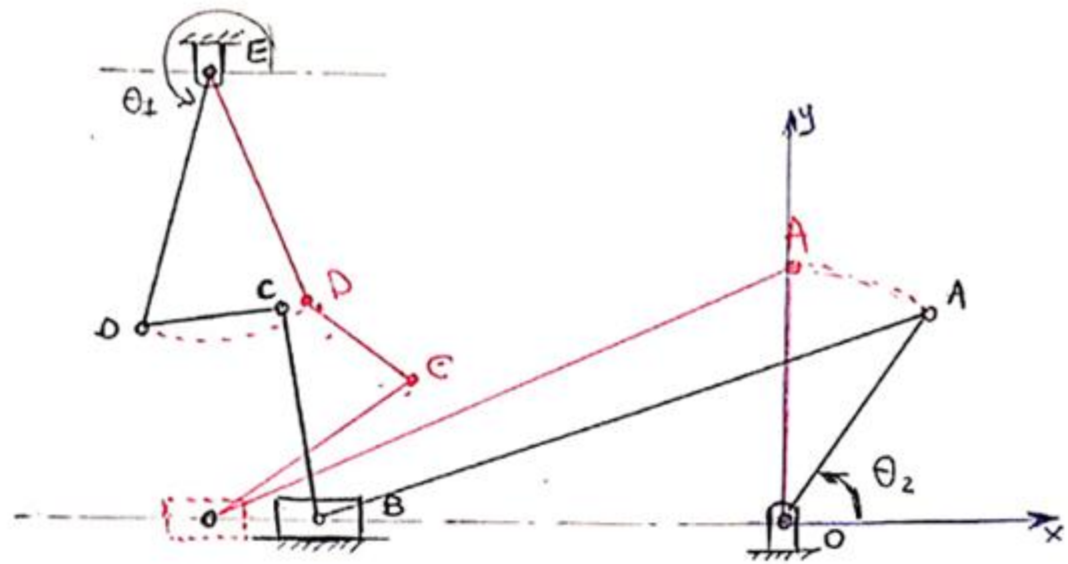
$W = 3 \cdot n - 2b - h$

$h = 0$

$b = 8$

$n = 7 - 1 = 6$

$\rightarrow W = 2$ ddL



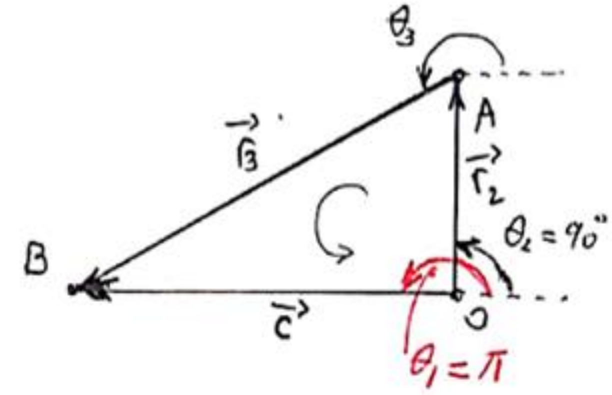
Cyclomatic number : $\gamma = L - N + 1 = 8 - 7 + 1 = 2$ cycles.

Note : $\gamma = 2$ *Loops* $\Rightarrow$ *2 closure loop equations are required to solve position problem*

The mechanism is stated at the position : $\theta_2 = 90°$, $\theta_1 = 300°$ (*In red*)

Loop I : $\underline{OABO}$

$$\vec{OA} + \vec{AB} = \vec{OB} \longrightarrow \vec{r_2} + \vec{r_3} = \vec{c}$$



$$r_2 e^{i\theta_2} + r_3 e^{i\theta_3} = c\, e^{i\pi}$$

$$\Rightarrow \begin{cases} r_2 \cos\theta_2 + r_3 \cos\theta_3 = -c \\ r_2 \sin\theta_2 + r_3 \sin\theta_3 = 0 \end{cases}$$

Unknowns : θ_3 *and* c

$$\theta_2 = 90° \longrightarrow \begin{cases} r_3 \cos\theta_3 = -c & (1) \\ r_2 + r_3 \sin\theta_3 = 0 & (2) \end{cases}$$

$$(2) \longrightarrow \sin\theta_3 = -\frac{r_2}{r_3} = -\frac{1}{3} \Rightarrow \begin{cases} \theta_3 = -19.47° \\ \theta_3 = 199.47° \end{cases}$$

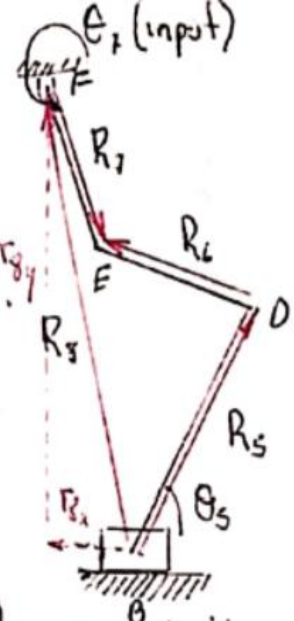
$\theta_3 = 199.47°$ → *Our configuration*

$$(1) \longrightarrow c = -r_3 \cos\theta_3 = -12 \times \cos(199.47) = 11{,}31 \text{ cm}.$$

$c = 11{,}31$ cm

Loop II:

$$R_8 + R_7 - R_6 - R_5 = 0$$



$r_{8x} = 12 - 11.3 = 0.7$

$r_{8y} = 8$

$r_5 = 5$

$r_6 = 3$

$r_7 = 4$

imag: $r_{8y} + r_7 \sin\theta_7 = r_5 \sin\theta_5 + r_6 \sin\theta_6$ ③ (θ_5 & θ_6 unknowns)

real: $r_{8x} + r_7 \cos\theta_7 = r_5 \cos\theta_5 + r_6 \cos\theta_6$ ④

let $\left.\begin{aligned} f &= r_{8y} + r_7 \sin\theta_7 \\ g &= r_{8x} + r_7 \cos\theta_7 \end{aligned}\right\}$ Known $\quad f = 4.536 \quad g = 1.3$

write eqs. ③ & ④ as follows (isolate $\sin\theta_6$ and $\cos\theta_6$), square both equations and add the together

$$+\begin{aligned} (f - r_5 \sin\theta_5)^2 &= (r_6 \sin\theta_6)^2 \\ (g - r_5 \cos\theta_5)^2 &= (r_6 \cos\theta_6)^2 \end{aligned}$$

$$f^2 + g^2 + r_5^2 - 2 f r_5 \sin\theta_5 - 2 g r_5 \cos\theta_5 = r_6^2$$

$a = 2 g r_5 = 13$

$b = 2 f r_5 = 45.36$

$c = f^2 + g^2 + r_5^2 - r_6^2 = 38.267$

hence,

$$\underbrace{2 g r_5}_{a} \cos\theta_5 + \underbrace{2 f r_5}_{b} \sin\theta_5 = \underbrace{f^2 + g^2 + r_5^2 - r_6^2}_{c} \implies a \cos\theta_5 + b \sin\theta_5 = c$$

$$\theta_5 = \operatorname{atan2}(b, a) \pm \operatorname{atan2}\left(\sqrt{a^2 + b^2 - c^2},\, c\right) \quad \begin{cases} \theta_5 = 109.82^\circ \\ \underline{\theta_5 = 38.19^\circ} \text{ (our configuration)} \end{cases}$$

From eqs ③ & ④ $\quad \sin\theta_6 = \dfrac{f - r_5 \sin\theta_5}{r_6}$ & $\cos\theta_6 = \dfrac{g - r_5 \cos\theta_5}{r_6} \implies \theta_6 = \operatorname{atan2}(\sin\theta_6, \cos\theta_6) = \underline{151.22^\circ}$