TD2 Descriptive Statistics and Probability (Solution)

Exercise 1.

(1) $\Omega = \{1\}$, $\mathcal{P}(\Omega) = \{\phi, \{1\}\}$.
(2) $\Omega = \{1, 2\}$, $\mathcal{P}(\Omega) = \{\phi, \{1\}, \{2\}, \{1, 2\}\}$.
(3) $\Omega = \{1, 2, 3\}$, $\mathcal{P}(\Omega) = \{\phi, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\}$.
(4) $\Omega = \{1, 2, 3, 4\}$, $\mathcal{P}(\Omega) = \{\phi, \{1\}, \{2\}, \{3\}, \{4\}, \{1, 2\}, \{1, 3\}, \{1, 4\}, \{2, 3\}, \{2, 4\}, \{3, 4\}, \{1, 2, 3\}, \{1, 2, 4\}, \{1, 3, 4\}, \{2, 3, 4\}, \{1, 2, 3, 4\}\}$.
(5) $\Omega = \phi$, $\mathcal{P}(\Omega) = \{\phi\}$.

Exercise 2.

(1) $(A \cup B) \cap (A \cup C) = A \cup (B \cap C)$.
(2) $(\overline{A} \cup B) \cap (A \cup B) = (\overline{A} \cap A) \cup B = \phi \cup B = B.$
(3) $(\overline{A} \cup B) \cap (A \cup B) \cap (\overline{A} \cup \overline{B}) = B \cap (\overline{A} \cup \overline{B}) = (B \cap \overline{A}) \cup (B \cap \overline{B}) = B \cap \overline{A}.$

Exercise 3.

(a) Each 8-symbol password is in the form

$$\begin{matrix} s_1 & s_2 & s_3 & s_4 & s_5 & s_6 & s_7 & s_8 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 66 & 66 & 66 & 66 & 66 & 66 & 66 & 66 \end{matrix}$$

So, from The Counting Principle ($C.P$) we have $\underbrace{66 \times 66 \times \cdots \times 66}_{n\ times} = 66^8$ symbol passwords

(b) Each 8-symbol passwords is an arrangement with repetition of 8 symbols among 66. Then there is 66^8 symbol password

Each choice of 2 letters among 26 is an arrangement with repetition, so we have 26^2possibilities, and each choice of 3 digits among 10 digits is an arrangement with repetition,

so we have 10^3 possibilities., then accrding to ($C.P$) we have $26^2 \times 10^3$ plates

If the repetitions are excluded, we are with an arrangement without repetition, there is A_{66}^8 possibilities for (a) and $A_{26}^2 \times A_{10}^3$ for (b).

Exercice 4.

We choose 6 digits with repetition from 10 digits, we have 10^6 possibilities

(a) If the number does not contain a 6, the telephone number takes the form $d_1d_2d_3d_4d_5d_6$, with $d_i \in \{1, \ldots 9\}$, i.e. there is 9 choices for each d_i, so there is 9^6 telephone numbers.

(b) There is 5^6 telephone number.

(c) The pattern 2345 can take 3 places, so there is 3×10^2 possibilities.

(d) Is the same as in (c).

Exercise 5.

a) pint, we have $4!$

b) proposition, we have $\dfrac{11!}{2!3!2!}$.

c) Mississippi, we have $\dfrac{11!}{4!4!2!}$.

d) arrangement, we have $\dfrac{11!}{2!2!2!2!}$.

Exercise 6.
We can form C_{23}^{4} committee.

Exercise 8.
(1)There is $C_{21}^{7}\times C_{14}^{7}\times\ C_{7}^{7}=\dfrac{21!}{7!7!7!}$ possibilites.
(2) a) If the distribution within offices has importance, there is 14!.
b) If the distribution within o¢ ces has no importance, thre is $C_{14}^{7}\times C_{7}^{7}=\ C_{14}^{7}$.

Exercise 9.
(1) $A\cup B\cup C$.
(2) $\left(A\cap\overline{B}\cap\overline{C}\right)\cup\left(\overline{A}\cap B\cap\overline{C}\right)\cup\left(\overline{A}\cap\overline{B}\cap C\right)\cup\left(\overline{A}\cap\overline{B}\cap\overline{C}\right)$.
(3) $\left(\overline{A}\cap\overline{B}\cap\overline{C}\right)=\left(\overline{A\cup B\cup C}\right)$.
(4) $A\cap B\cap C$.
(5) $\left(A\cap\overline{B}\cap\overline{C}\right)\cup\left(\overline{A}\cap B\cap\overline{C}\right)\cup\left(\overline{A}\cap\overline{B}\cap C\right)$
(6) $A\cap B\cap\overline{C}$.
(7) $A\cup\overline{B}$.

Exercise 10.
$E=\{$at least one event occurs$\}=A\cup B$, then

$$P\left(E\right)=P\left(A\right)+P\left(B\right)-P\left(A\cap B\right)=\frac{1}{4}+\frac{2}{3}-\frac{1}{8}=\frac{19}{24}.$$

$F=\{$only one event occurs$\}=\left(A\cap\overline{B}\right)\cup\left(\overline{A}\cap B\right)$. $\left(A\cap\overline{B}\right)$ and $\left(\overline{A}\cap B\right)$ are a disjoint events, then

$$P\left(F\right)=P\left(A\cap\overline{B}\right)+P\left(\overline{A}\cap B\right)$$

$A=A\cap\left(B\cup\overline{B}\right)=\left(A\cap B\right)\cup\left(A\cap\overline{B}\right)\Rightarrow P\left(A\cap\overline{B}\right)=P\left(A\right)-P\left(A\cap B\right)=\dfrac{1}{8}$.
$B=\left(A\cup\overline{A}\right)\cap B=\left(A\cap B\right)\cup\left(\overline{A}\cap B\right)\Rightarrow P\left(\overline{A}\cap B\right)=P\left(B\right)-P\left(A\cap B\right)=\dfrac{13}{24}$.
Finally

$$P\left(F\right)=\frac{1}{8}+\frac{13}{24}=\frac{2}{3}.$$

Exercise 11.
The urn contains 7 red and 5 blue balls, we take out one by one the balls, then we have

$$\left|\Omega\right|=C_{12}^{5}\times C_{7}^{7}=\frac{12!}{5!7!}.$$

(a) Let B, the first ball is blue, then

$$P\left(B\right)=\frac{\left|B\right|}{\left|\Omega\right|}=\frac{C_{11}^{4}}{C_{12}^{5}}.$$

(b) Let C,the last ball is blue (like as a), we fix the place of the blue ball at last, the rest of balls is 11, then we have

$$P\left(C\right)=\frac{\left|C\right|}{\left|\Omega\right|}=\frac{C_{11}^{4}}{C_{12}^{5}}.$$

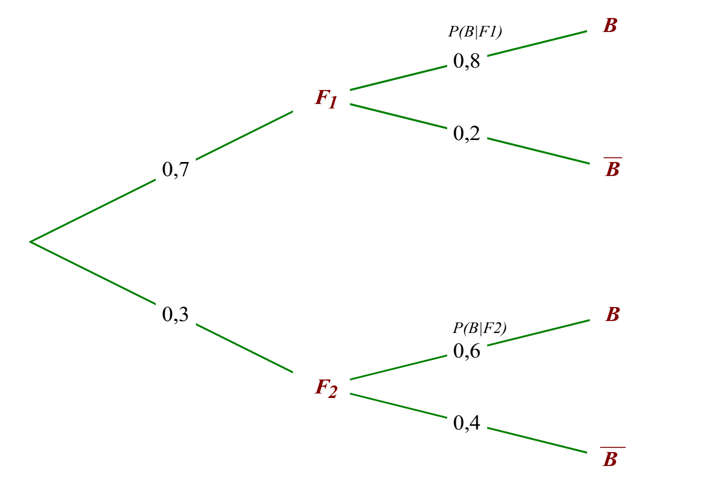
(c) Now $P\left(C|B\right)=\dfrac{P\left(C\cap B\right)}{P\left(B\right)}=\dfrac{\left|C\cap B\right|}{\left|B\right|}$,

$$\left|C\cap B\right|=\frac{10!}{3!7!}=C_{10}^{3},$$

then

$$P(C|B) = \frac{C_{10}^{3}}{5/12}.$$

Exercise 12.
Let F_1 : The ampoule come from factoty 1
F_2 : The ampoule come from factoty 2,
and B : The ampoule is type type B
$\overline{B}$: The ampoule is not type type B.
We have $P(F_1) = 0,7$ and $P(F_2) = 0,3$.
Consider rhe following tree



(1)

$$P(B) = P(B|F_1)P(F_1) + P(B|F_2)P(F_2) = 0,8 \times 0,7 + 0,6 \times 0,3 = 0,74.$$

(2) $P(F_1|B)$, we use Bayes's Theorem

$$P(F_1|B) = \frac{P(F_1 \cap B)}{P(B)} = \frac{P(B|F_1)P(F_1)}{P(B)} = \frac{0,56}{0,74} = 0,76.$$

Exercise 13.
$\Omega = \{1,2,3,4,5,6\}$, $A = \{1,2,5,6\}$, $B = \{2,3\}$, $A \cap B = \{2\}$
Relative to P

$$\begin{array}{l} P(A) = P(1) + P(2) + P(5) + P(6) = \frac{3}{10} + \frac{1}{5} + \frac{1}{20} + \frac{1}{4} = \frac{4}{5}, \\ P(B) = P(2) + P(3) = \frac{1}{5} + \frac{1}{20} = \frac{1}{4}, \\ P(A) \times P(B) = \frac{1}{5}, \\ P(A \cap B) = \frac{1}{5} = P(A) \times P(B). \end{array}$$

Then relative to P, A and B are independent.
Relative to P'

$$\begin{array}{l} P(A) = \frac{|A|}{|\Omega|} = \frac{2}{3} \\ P(B) = \frac{|B|}{|\Omega|} = \frac{1}{3}, \\ P(A) \times P(B) = \frac{2}{9}, \\ P(A \cap B) = \frac{|A \cap B|}{|\Omega|} = \frac{1}{6} \neq P(A) \times P(B). \end{array}$$

Then relative to P', A and B are not independent.