Homework

Exercise 1

Let

$$W = \{(x, y, z) \in \mathbb{R}^3 \mid x + y + z = 0\}$$

1) Show that W is a vector subspace of $\mathbb{R}^3$
2) Find a basis of W
3) Determine the dimension of W

Exercise 2

Let

$$W = \{(x, y, z) \in \mathbb{R}^3 \mid x \ - 2y + z = 0\}$$

1) Prove that W is a subspace of $\mathbb{R}^3$
2) Find a basis of W.
3) Find dim (W).

Exercise 3

Let

$$W = \{(x, y, z, t) \in \mathbb{R}^4 \mid x + y + z + t = 0\}$$

1) Show that W is a subspace of $\mathbb{R}^4$
2) Determine a basis of W
3) Find the dimension of W.

Exercise 4

Consider the vectors in $\mathbb{R}^3$

$$V_1 = (1, 0, 1), \qquad V_2 = (2, 1, 3), \qquad V_3 = (0, 1, 1)$$

1) Are the vectors linearly independent?
2) Do they form a basis of $\mathbb{R}^3$?
3) Find the dimension of the vector space generated by these vectors.

Exercise 5

Let

$$V_1 = (1, 1, 0), \qquad V_2 = (2, 1, 1), \qquad V_3 = (3, 2, 1)$$

1) Determine whether the vectors are linearly independent.
2) Find a basis of the space they generate.
3) Determine the dimension.

Exercise 6

Let

$$f : \mathbb{R}^2 \to \mathbb{R}^2$$

defined by

$$f(x, y) = (x + 2y,\, 3x + y)$$

1) Verify that f is a linear application
2) Find the kernel $Ker(f)$
3) Find the image $Im(f)$
4) Determine $\dim(Ker(f))$ and $\dim(Im(f))$.

Exercise 7

Let

$$T : \mathbb{R}^3 \to \mathbb{R}^2$$

defined by

$$T(x, y, z) = (x + y,\, y + z)$$

1) Find the kernel of T
2) Find the image of T
3) Determine a basis for the kernel.
4) Determine a basis for the image.
5) Verify the Rank-Nullity theorem.

Exercise 8

Let

$$T : \mathbb{R}^3 \to \mathbb{R}^3$$

$$T(x, y, z) = (x - y,\, y - z,\, x - z)$$

1) Show that T is linear.
2) Determine Ker(T).
3) Determine Im(T).
4) Find a basis of the kernel.
5) Find a basis of the image.
6) Find the rank of T

Exercise 9

Let

$$A = \begin{pmatrix} 2 & -1 \\ 3 & 4 \end{pmatrix}$$

,

$$B = \begin{pmatrix} 1 & 5 \\ -2 & 0 \end{pmatrix}$$

1) Compute:
a) $A + B$
b) $A - B$
c) $2A$
2) Compute the matrix product $A \times B$

3) Verify whether $A \times B = B \times A$

Exercise 10

Let

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & -1 & 4 \end{pmatrix}$$

1) Determine the transpose matrix A^t

2) Compute $(A^t)^t$

3) Verify the property:

$$(A+B)^t = A^t + B^T$$

for suitable matrices A and B

Exercise 11

Let

$$A = \begin{pmatrix} 3 & 1 \\ 2 & 5 \end{pmatrix}$$

1) Compute the determinant of matrix A

2) Determine whether matrix A is invertible.

3) Find A^{-1} if it exists.

Exercise 12

Let

$$A = \begin{pmatrix} 1 & 2 & 0 \\ -1 & 3 & 1 \\ 2 & 1 & 4 \end{pmatrix}$$

1) Compute the determinant of A

2) Determine whether the matrix is invertible.

Exercise 13

Let

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 1 & 1 & 1 \end{pmatrix}$$

1) Reduce matrix A using elementary row operations.

2) Determine the rank of matrix A

Exercise 14

Let

$$A = \begin{pmatrix} 4 & 6 \\ 2 & 7 \end{pmatrix}$$

1) Compute the determinant of A

2) Find the inverse matrix A^{-1}

3) Verify that $AA^{-1} = I$

where I is the identity matrix.