TD 01

Exercise 1

1. Consider in $\mathbb{R}^3$ the subset F defined by :

$$F = \{(x, y, z) \in \mathbb{R}^3 \mid 2x + y - z = 0\}.$$

   Show that F is a subspace of $\mathbb{R}^3$.

2. Give a basis for F, and what is its dimension ?
3. Is F equal to $\mathbb{R}^3$?

Exercise 2

Consider in $\mathbb{R}^3$ the subset F defined by :

$$F = \{(x - y, 2x + y + 4z, 3y + 2z) \mid x, y, z \in \mathbb{R}\}.$$

1. Show that F is a subspace of $\mathbb{R}^3$.
2. Give a basis for F, and what is its dimension ?
3. Is F equal to $\mathbb{R}^3$?

Exercise 3

Consider in $\mathbb{R}^4$ the subset F defined by :

$$F = \{(x, y, z, t) \in \mathbb{R}^4 \mid (x + z = 0) \wedge (y + t = 0)\}.$$

1. Show that F is a subspace of $\mathbb{R}^4$.
2. Give a basis for F, and deduce its dimension.

Exercise 4

1. Show that the family $\{(1, 2), (-1, 1)\}$ generates $\mathbb{R}^2$.
2. Which families are free among the following : $F_1 = \{(1, 1, 0), (1, 0, 0), (0, 1, 1)\}$, $F_2 = \{(0, 1, 1, 0), (1, 1, 1, 0), (2, 1, 1, 0)\}$?
3. Show that the family $\{(1, 2), (-1, 1)\}$ is a basis for $\mathbb{R}^2$, and that the family $F_1 = \{(1, 1, 0), (1, 0, 0), (0, 1, 1)\}$ is a basis for $\mathbb{R}^3$.