# Tutorial n°2: Electrostatics

## Part-II: Discrete charge distribution

### Exercise 1

Find the total charge Q for the following distributions:

1- Linear charge distributed uniformly on a ring of radius a
2- Surface charge distributed uniformly on a sphere of radius R.
3- Volume charge distributed uniformly on a cylinder of radius R and height h.
4- Volume charge distributed uniformly in a sphere of radius R.

### Exercise 2

We consider a wire of length **2L**, of negligible diameter, uniformly charged with a positive and constant linear charge density $\boldsymbol{\lambda}$.

1- Determine the total charge $Q$ of the wire.
2- Determine the electric field $E$ created by this wire at a point M located on its axis of symmetry Oy: ($\overrightarrow{OM} = b\,\vec{j}$).
3- Deduce the electric field created by an infinite wire.

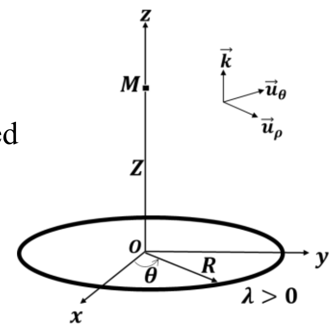
### Exercise 3

We consider a disk of radius $\boldsymbol{R}$, uniformly charged with a positive and constant surface charge density $\boldsymbol{\sigma}$.

1- Determine the total charge $Q$ of the disk.
2- Determine the electric field $E$ created by this disk at a point $N$ located on its axis of symmetry Ox: ($\overrightarrow{ON} = a\,\vec{i}$).
3- Deduce the electric field created by an infinite plane.

### Exercise 4

A ring (circular wire) with center O, radius R and axis Oz, is uniformly charged with a positive and constant linear charge density $\boldsymbol{\lambda}$. Give the expression of:




1- The total charge Q of the ring.
2- The electrostatic field created by the ring at a point M located on its axis of symmetry Oz. Deduce the electrostatic field at point O.
3- The electrostatic potential at point M.
4- The electrostatic force exerted by the ring on a punctual charge $\boldsymbol{q}$, placed at $M$.

### Exercise 5

We consider an infinite wire, uniformly charged by a positive and constant linear charge density ($\lambda > 0$).

1- Calculate the total charge Q carried by the infinite wire.
2- Determine the expression of the electric field $E(r)$ in all point $M$ of space.
3- Graph (draw) the curve of $E(r)$

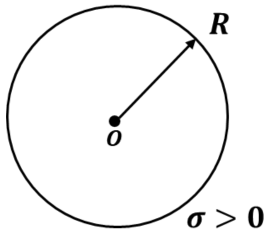
## Exercise 6

We consider an infinite plane, uniformly charged by a positive and constant surface charge density $(\sigma > 0)$.

1- Calculate the total charge Q carried by the infinite plane.
2- Determine the expression of the electric field $E(r)$ in all point $M$ of space.
3- Graph (draw) the curve of $E(r)$

## Exercise 7

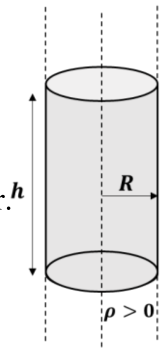
A hollow sphere with center $O$ and radius $R$ is uniformly charged by a positive and constant surface charge density $\sigma$ $(\sigma > 0)$.




1- Calculate the total charge Q of the sphere.
2- Determine the expression of the electric field $E(r)$ at any point $M$ in space.
3- Deduce the expression of the electric potential $V(r)$.
4- Graph (draw) the variation curve of $E(r)$ and $V(r)$. Is there continuity?

## Exercise 8

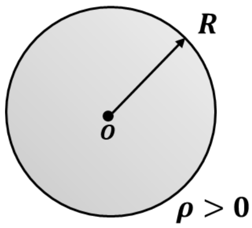
Let $C$ be a solid cylinder of the axis of revolution $(Oz)$, of radius $R$ and of infinite length, uniformly charged by a positive and constant volume charge density ρ.




1- Determine the electrostatic field inside, on the surface and outside of the cylinder. Check that the electric field is continuous at the surface of the cylinder.

## Exercise 9

A full sphere with center $O$ and radius $R$ is uniformly charged by a positive and constant volume charge density $\rho$ $(\rho > 0)$.




1- Calculate the total charge Q of the sphere.
2- Determine the expression of the electric field $E(r)$ at any point $M$ in space.
3- Deduce the expression of the electric potential $V(r)$.
4- Graph (draw) the variation curve of $E(r)$ and $V(r)$. Is there continuity?

We give: $\int \frac{x\,dx}{\sqrt{x^2+y^2}^3} = \frac{-1}{\sqrt{x^2+y^2}}$ ; $\int \frac{dx}{\sqrt{x^2+y^2}^3} = \frac{x}{y^2\sqrt{x^2+y^2}}$

## Exo- 1

**The total charge Q**

1- **For a ring**

$Q = \int dq = \int \lambda \, dl = \int_0^{2\pi} \lambda \, r d\theta$ ; $(dl = r d\theta)$

$\Rightarrow \boldsymbol{Q = 2\pi a \lambda}$

2- **For a sphere**

$Q = \int dq = \int \sigma \, dS$ ; $(dS = r\, d\theta \, r \sin\theta \, d\varphi)$

$= \sigma r^2 \int_0^{\pi} \sin\theta \, d\theta \int_0^{2\pi} d\varphi \Rightarrow \boldsymbol{Q = 4\pi R^2 \sigma}$

3- **For a cylinder**

$Q = \int dq = \int \rho \, dV$ ; $(dV = dr\, r\, d\theta \, dz)$

$= \rho \int_0^R r\, dr \int_0^{2\pi} d\theta \int_0^h dz = \rho \left[\frac{r^2}{2}\right]_0^R [\theta]_0^{2\pi} [z]_0^h$

$\Rightarrow \boldsymbol{Q = \pi R^2 h \rho}$

4- **For a sphere**

$Q = \int dq = \int \rho \, dV$ ; $(dV = dr\, r\, d\theta \, r \sin\theta \, d\varphi)$

$= \rho \int_0^R r^2 \, dr \int_0^{\pi} \sin\theta \, d\theta \int_0^{2\pi} d\varphi \Rightarrow \boldsymbol{Q = \frac{4}{3}\pi R^3 \rho}$

**Direct Method:**

1- $Q_{ring} = \int dq = \int \lambda \, dl = \lambda \, l \Rightarrow \boldsymbol{Q = 2\pi a \lambda}$

2- $Q_{sph} = \int dq = \int \sigma \, dS = \sigma \, S \Rightarrow \boldsymbol{Q = 4\pi R^2 \sigma}$

3- $Q_{cyl} = \int dq = \int \rho \, dV = \rho V \Rightarrow \boldsymbol{Q = \pi R^2 h \rho}$

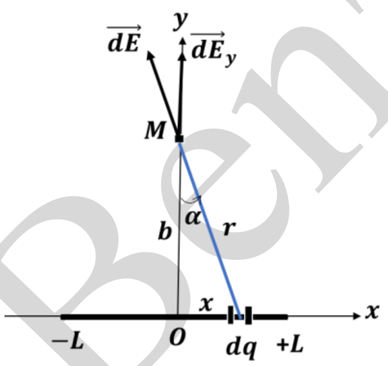
4- $Q_{sph} = \int dq = \int \rho \, dV = \rho V \Rightarrow \boldsymbol{Q = \frac{4}{3}\pi R^3 \rho}$

## Exo-2

**1) The total charge Q of the wire**

We have: $Q = \int dq = \int_{-l}^{+l} \lambda \, dl = 2\lambda \, l \Rightarrow \boldsymbol{Q = 2\lambda l}$

**2) The electrostatic field at point M.**




According to the figure, by projecting the vector $(\overrightarrow{dE})$ onto the axes, we obtain:

$$\overrightarrow{dE} = dE_x \, \vec{\imath} + dE_y \, \vec{\jmath}$$

Here, the system presents a symmetry of revolution around the axis Oy which means that: $E_x = 0$

So: $\vec{E} = E_y \, \vec{\jmath}$

By projection on the Oy axis: $dE_y = dE \cos\alpha$ ... $(*)$

Knowing that:

$$\begin{cases} dE = \dfrac{k\, dq}{r^2} \\ dq = \lambda \, dl = \lambda dx \\ r = \sqrt{b^2 + x^2} \\ \cos\alpha = \dfrac{y}{\sqrt{b^2 + x^2}} \end{cases}$$ , we replace them in eq$(*)$

$$\Rightarrow dE_y = k\, \lambda \, b \frac{dx}{\sqrt{b^2 + x^2}^3}$$

By integration:

$$\Rightarrow E_y = k\, \lambda \, b \int_{-l}^{+l} \frac{dx}{\sqrt{b^2 + x^2}^3} \Rightarrow \boldsymbol{E_M = \frac{2k\, \lambda \, l}{b\sqrt{b^2 + l^2}}}$$

$$\boldsymbol{Or} \quad : \boldsymbol{\overrightarrow{E_M} = \frac{2k\, \lambda \, l}{b\sqrt{b^2 + l^2}} \, \vec{\jmath}}$$

**The electric field created by an infinite wire.**

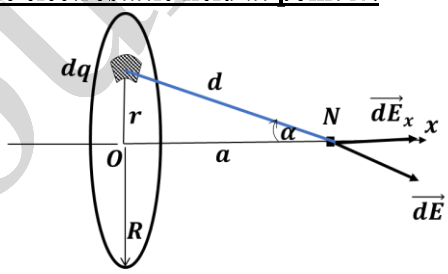
Infinite wire $\Rightarrow \lim_{l \to \infty} E_M$: $\quad \boldsymbol{E = \frac{2k\,\lambda}{b} = \frac{\lambda}{2\pi\varepsilon_0 \, b}}$

## Exo-3

**1) The total charge Q of the disc**

We have: $Q = \iint dq = \iint \lambda \, dS = \lambda \, S \Rightarrow \boldsymbol{Q = \pi R^2 \sigma}$

**2) The electrostatic field at point *N*.**




According to the figure, by projecting the vector $\overrightarrow{dE}$ onto the axes, we obtain:

$$\overrightarrow{dE} = dE_x \, \vec{\imath} + dE_y \, \vec{\jmath}$$

Here, the system presents a symmetry of revolution around the axis Ox which means that: $E_y = 0$

So: $\vec{E} = E_x \, \vec{\imath}$

By projection into Ox axis: $dE_x = dE \cos\alpha$ ... $(*)$

Knowing that:

$$\begin{cases} dE = \dfrac{k\, dq}{r^2} \\ dq = \sigma \, dS \\ dS = r\, dr\, d\theta \\ d = \sqrt{a^2 + r^2} \\ \cos\alpha = \dfrac{y}{\sqrt{a^2 + r^2}} \end{cases}$$ , we replace them in eq $(*)$

$$\Rightarrow dE_y = k\, \sigma \, a \frac{r\, dr\, d\theta}{\sqrt{a^2 + r^2}^3}$$

By integration:

$$\Rightarrow E_y = k\,\sigma\,a \int_0^R \frac{r\,dr}{\sqrt{a^2+r^2}^3}\int_0^{2\pi} d\theta$$

$$\Rightarrow \boldsymbol{E_N} = \frac{\boldsymbol{\sigma}}{\boldsymbol{2\,\varepsilon_0}}\left[\boldsymbol{1} - \frac{\boldsymbol{1}}{\sqrt{\boldsymbol{a^2+R^2}}}\right]$$

$$\boldsymbol{Or} \quad : \overrightarrow{\boldsymbol{E_N}} = \frac{\boldsymbol{\sigma}}{\boldsymbol{2\,\varepsilon_0}}\left[\boldsymbol{1} - \frac{\boldsymbol{1}}{\sqrt{\boldsymbol{a^2+R^2}}}\right]\vec{\imath}$$

**The electric field created by an infinite plane.**

Infinite plane $\Rightarrow \lim_{R\to\infty} E_M$: $\quad \boldsymbol{E} = \frac{\boldsymbol{\sigma}}{\boldsymbol{2\,\varepsilon_0}}$

## Exo-4:

**Symmetry of the problem**: the distribution presents a symmetry of axial revolution along Oz. Any plane containing the Oz axis is a plane of symmetry.

So: $\overrightarrow{E_M} = E\,\vec{k}$.

**Invariance**: by rotation around Oz, E does not depend on $\theta$. So: $\vec{E} = \vec{E}(z)$ .

### 1) The total charge Q of ring

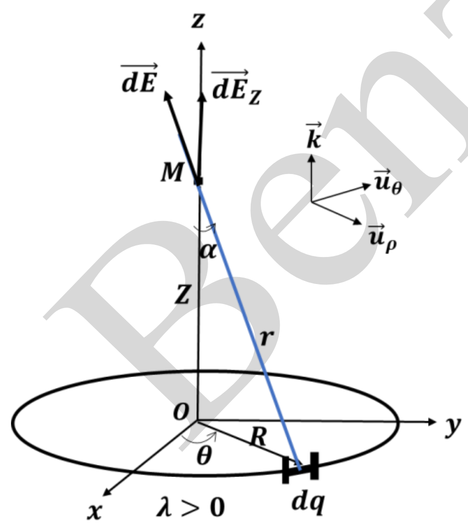
We have: $Q = \int dq = \int \lambda\, dl = \lambda\, l_{cercle}$

$\Rightarrow \boldsymbol{Q = 2\pi R\lambda}$

### 2) The electrostatic field at point M.

According to the figure, by projecting the vector $\overrightarrow{dE}$ onto the axes, we obtain:

$$\overrightarrow{dE} = dE_\rho\,\overrightarrow{u_\rho} + dE_\theta\,\overrightarrow{u_\theta} + dE_z\,\vec{k}$$




From symmetry: $E_\rho = E_\theta = 0$

So: $\vec{E} = E_z\,\vec{k}$

By projection onto Oz axis: $dE_z = dE\cos\alpha$ … … (∗)

knowing that:

$$\begin{cases} dE = \dfrac{k\,dq}{r^2} \\ dq = \lambda\, dl \\ dl = R\, d\theta \\ r = \sqrt{Z^2+R^2} \\ \cos\alpha = \dfrac{Z}{\sqrt{Z^2+R^2}} \end{cases}$$

, we remplace them in eq(∗)

$$\Rightarrow dE_Z = \frac{k\,\lambda\,R\,Z}{\sqrt{Z^2+R^2}^3}\,d\theta$$

by projection:

$$\Rightarrow E_Z = \frac{k\,\lambda\,R\,Z}{\sqrt{Z^2+R^2}^3}\int_0^{2\pi} d\theta = \frac{\lambda\,R\,Z}{2\,\varepsilon_0\sqrt{Z^2+R^2}^3}$$

$$\Rightarrow \overrightarrow{\boldsymbol{E_M}} = \frac{\boldsymbol{\lambda\,R\,Z}}{\boldsymbol{2\,\varepsilon_0\sqrt{Z^2+R^2}^3}}\,\vec{\boldsymbol{k}}$$

### The electrostatic field at point O.

At point O: $Z = 0 \Rightarrow \overrightarrow{\boldsymbol{E_O}} = \vec{\boldsymbol{0}}$

### 3) The electrostatic potential at point M

To deduce the electric potential V, we use the formula $\vec{E} = -\overrightarrow{grad}\,V$. Since the field is axial along Oz, we can write:

$$E = -\frac{dV}{dZ} \Rightarrow V = -\int E\,dZ$$

$$\Rightarrow V = -\frac{\lambda\,R}{2\,\varepsilon_0}\int \frac{Z}{\sqrt{Z^2+R^2}^3}\,dZ$$

$$\Rightarrow V = \frac{\lambda\,R}{2\,\varepsilon_0\sqrt{Z^2+R^2}} + C^t$$

To determine C, we use the limit conditions of the potential: $\lim_{r\to\infty} V = 0$

$$\Rightarrow \lim_{r\to\infty} V = 0 + \mathrm{C} = 0 \Rightarrow \boldsymbol{C = 0}$$

$$\Rightarrow \boldsymbol{V} = \frac{\boldsymbol{\lambda\,R}}{\boldsymbol{2\,\varepsilon_0\sqrt{Z^2+R^2}}}$$

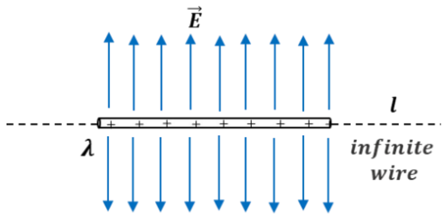
### 4) The electrostatic force

$\vec{F_e} = q\,\vec{E}$

$$\overrightarrow{\boldsymbol{F_e}}(\boldsymbol{M}) = \frac{\boldsymbol{\lambda\,R\,Z\,q}}{\boldsymbol{2\,\varepsilon_0\sqrt{Z^2+R^2}^3}}\,\vec{\boldsymbol{k}}$$

## Exo-5

By a symmetry, the field $\vec{\boldsymbol{E}}$ at any point ($\boldsymbol{r}$) in space is radial, perpendicular to the axis of the wire, and directed towards the exterior.



1) **The total charge $Q_T$**

$Q_T = \int dq = \int_0^l \lambda\, dl = \lambda l \qquad \Rightarrow \boldsymbol{Q_T = \lambda l}$

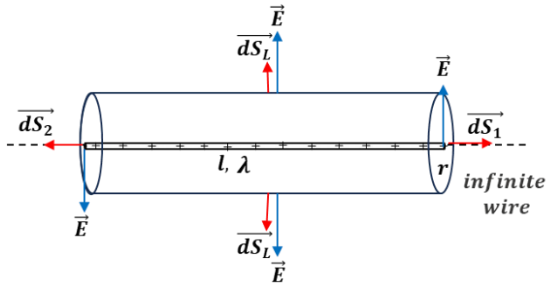
2) **The field $E$ in all points of space**

According to Gauss's theorem:

$$\boldsymbol{\phi = \oiint \vec{E}\,.\,\overrightarrow{dS_G} = \frac{Q_{int}}{\varepsilon_0}}$$

The Gaussian surface which is suitable here is a surface of a cylinder $(r, h)$, where:

- The axis of the cylinder coincides with the wire.
- The two bases of the cylinder cover the extremities of the wire.




According to the diagram, there are three surfaces:

1- Base surface $S_1$ (disk) of $\overrightarrow{dS_1}$
2- Base surface $S_2$ (disk) of $\overrightarrow{dS_2}$
3- Lateral surface $S_L$ (rectangular) of $\overrightarrow{dS_L}$

The total flux through all surfaces constituting the Gaussian cylinder is given by:

$$\emptyset_T = \emptyset_1 + \emptyset_2 + \emptyset_L$$

Where:

$$\begin{cases} \emptyset_1 = \iint \vec{E}.\overrightarrow{dS_1} = \iint E.dS_1.\cos\theta = 0; & (\vec{E} \perp \overrightarrow{dS_1}) \\ \emptyset_2 = \iint \vec{E}.\overrightarrow{dS_2} = \iint E.dS_2.\cos\theta = 0; & (\vec{E} \perp \overrightarrow{dS_2}) \\ \emptyset_L = \iint \vec{E}.\overrightarrow{dS_L} = \iint E.dS_L.\cos\theta = E.S_L; & (\vec{E} \parallel \overrightarrow{dS_L}) \end{cases}$$

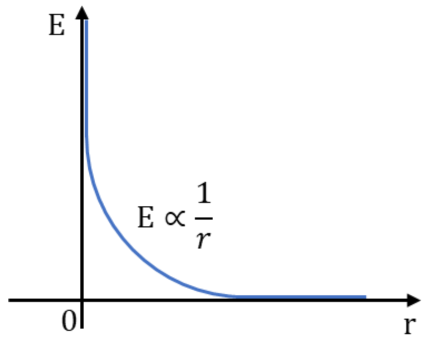
Therefore:

$$\emptyset_T = E.S_L = \frac{Q}{\varepsilon_0}$$

Knowing that: $\begin{cases} S_L = 2\pi r h \\ Q_T = \lambda\, l \\ h = l \end{cases}$

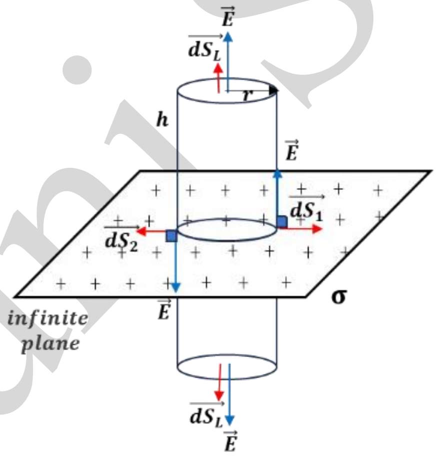
$$\boldsymbol{E(r) = \frac{\lambda}{2\pi\, \varepsilon_0\, r}}$$

3) **Curve $E(r)$**




**Exo-6**

By a symmetry, the field $E$ at all points in space is: radial, perpendicular to the plane and directed towards the exterior.




1) **The total charge $Q_T$**

$Q_T = \int dq = \int \sigma\, dS = \sigma\, S \qquad \Rightarrow \boldsymbol{Q_T = \sigma\, S}$

2) **The field $E$ at all points in the space**

According to Gauss's theorem:

$$\boldsymbol{\phi = \oiint \vec{E}\,.\,\overrightarrow{dS_G} = \frac{Q_{int}}{\varepsilon_0}}$$

The most suitable Gauss surface $\boldsymbol{S_G}$ which gives a high degree of symmetry is a cylinder of $(r, h)$.

The total flux through all surfaces constituting the Gaussian cylinder is given by:

$$\emptyset_T = \emptyset_1 + \emptyset_2 + \emptyset_L$$

Where:

$$\begin{cases} \emptyset_1 = \iint \vec{E}.\overrightarrow{dS_1} = \iint E.dS_1.\cos\theta = E.S; & (\vec{E} \parallel \overrightarrow{dS_1}) \\ \emptyset_2 = \iint \vec{E}.\overrightarrow{dS_2} = \iint E.dS_2.\cos\theta = E.S; & (\vec{E} \parallel \overrightarrow{dS_2}) \\ \emptyset_L = \iint \vec{E}.\overrightarrow{dS_L} = \iint E.dS_L.\cos 90° = 0; & (\vec{E} \perp \overrightarrow{dS_L}) \end{cases}$$

Therefore:

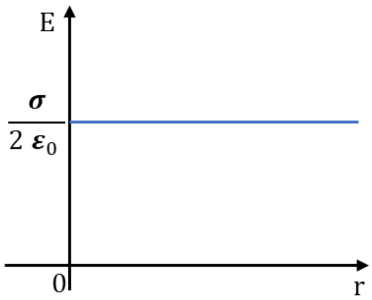
$\phi_T = 2E.S = \frac{Q_{int}}{\varepsilon_0}$

$Q_{int}$ : here presents the charge of the plane inside the cylinder (a disk of radius $r$).

$$\begin{cases} Q_{int} = \sigma\, \pi\, r^2 \\ S = \pi\, r^2 \end{cases}$$

$$\boldsymbol{E(r) = \frac{\sigma}{2\, \varepsilon_0}}$$

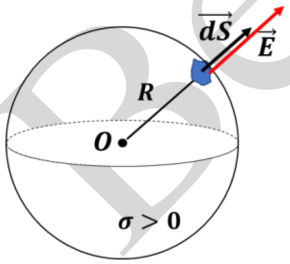
**3) <u>Curve $E(r)$</u>**




**<u>Exo-7</u>**

The field has spherical symmetry around the center of the sphere O, under any rotation (θ, φ) the field remains the same. By reason of symmetry, the electrostatic field created by a sphere is:

- Radial $\vec{E} = E(r)\,\vec{u_r}$
- Depends only on the radius $r$ : $E(r)$.




**<u>1) The total charge Q</u>**

We have: $Q = \iint dq = \iint \sigma\, dS = \sigma\, S_{sphére}$

$\Rightarrow \boldsymbol{Q = 4\pi R^2 \sigma}$

**<u>2) The electric field $E(r)$</u>**

To determine the field at any point M in space, we use Gauss' theorem:

$$\boldsymbol{\phi = \oiint \vec{E}\,.\,\overrightarrow{dS_G} = \frac{Q_{int}}{\varepsilon_0}}$$

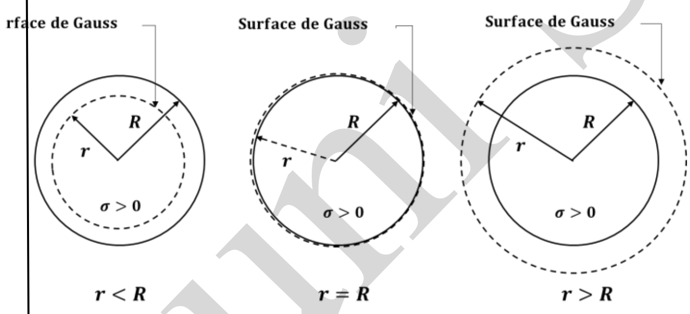
As the electrostatic field is radial, $\vec{E}$ and $\overrightarrow{dS}$ are collinear ($\vec{E} \parallel \overrightarrow{dS} \Rightarrow \theta = 0°$, we can then write:

$$\Rightarrow \oiint E.dS_G \cos\theta = \frac{Q_{int}}{\varepsilon_0}; (\vec{E} \parallel \overrightarrow{dS} \Rightarrow \theta = 0°)$$

$$\Rightarrow \boldsymbol{E.S_G = \frac{Q_{int}}{\varepsilon_0}} \ldots (*)$$

Due to spherical symmetry, here the suitable closed Gauss surface $S_G$ is a surface of a sphere with center O and radius $r$. ($S_G = 4\pi r^2$).

To determine the field at any point M in space, we distinguish 03-regions:




**<u>Region-1 $r < R$ :</u>**

In this case the Gauss surface does not contain any electric charge

$$\begin{cases} S_G = 4\pi r^2 \\ Q_{int} = 0 \end{cases} \Rightarrow l'eq(*) \Rightarrow \boldsymbol{E_1 = 0}$$

**<u>Region-2 $r = R$ :</u>**

In this case all the charge ported by the sphere is inside the Gaussian surface.

$$\begin{cases} S_G = 4\pi r^2 \\ Q_{int} = Q \end{cases} \Rightarrow l'eq(*) \Rightarrow E_2 4\pi r^2 = \frac{4\pi R^2 \sigma}{\varepsilon_0}$$

$$r = R \Rightarrow \boldsymbol{E_2 = \frac{\sigma}{\varepsilon_0}}$$

**<u>Region-3 r > R :</u>**

Likewise, all the charge ported by the sphere is

inside the Gaussian surface

$$\begin{cases} S_G = 4\pi r^2 \\ Q_{int} = Q \end{cases} \Rightarrow l'eq(*) \Rightarrow E_3 4\pi r^2 = \frac{4\pi R^2 \sigma}{\varepsilon_0}$$

$$\Rightarrow \boldsymbol{E_3 = \frac{\sigma R^2}{\varepsilon_0 r^2}}$$

**<u>Note:</u>**

We can obtain the expression of the field $E_2$ on the surface of the sphere ($r = R$), by replacing with R in the expression of $E_3$ found outside the sphere.

**<u>2) The Electric potential $V(r)$</u>**

To determine the potential, we use the relation:

$\vec{E} = -\overrightarrow{grad}\, V$

E is radial $\Rightarrow E = -\frac{dV}{dr} \Rightarrow V = -\int E\, dr$

**<u>Region-1 $r < R$ :</u>**

$$V_3 = -\int E_3\, dr = V = -\int \frac{\sigma R^2}{\varepsilon_0 r^2}\, dr = \frac{\sigma R^2}{\varepsilon_0 r} + C_3$$

For determining $C_3$ , we use the limit conditions of the potential: $\lim_{r \to \infty} V = 0$

$$\Rightarrow \lim_{r \to \infty} V_3 = 0 + C_3 = 0 \Rightarrow \mathbf{C_3 = 0}$$

$$\boldsymbol{V_3 = \frac{\sigma R^2}{\varepsilon_0 r}}$$

**Region-2 $r = R$ :**

$$V_2 = -\int E_2\, dr = -\int \frac{\sigma}{\varepsilon_0}\, dr = -\frac{\sigma}{\varepsilon_0} r + C_2$$
$$= -\frac{\sigma}{\varepsilon_0} R + C_2$$

To determine $C_2$, we use the condition of continuity of the potential at the interface $r = R$ :

$V_3(r = R) = V_2(r = R)$

$$\begin{cases} V_2(R) = -\frac{\sigma}{\varepsilon_0} R + C_2 \\ V_3(R) = \frac{\sigma R}{\varepsilon_0} \end{cases} \Rightarrow C_2 = \frac{2\sigma R}{\varepsilon_0}$$

$$\Rightarrow \boldsymbol{V_2 = \frac{\sigma}{\varepsilon_0} R}$$

**Region-3 r > R :**

$$V_1 = -\int E_1\, dr = -\int 0\, dr = C_1$$

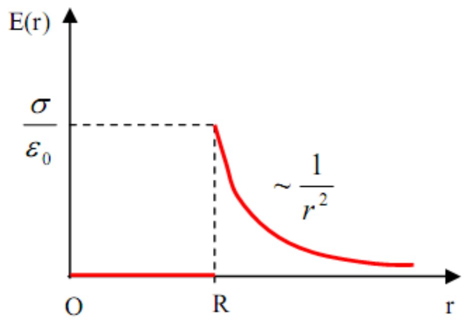
To determine $C_1$, we use the condition of continuity of the potential at the interface $R$ :

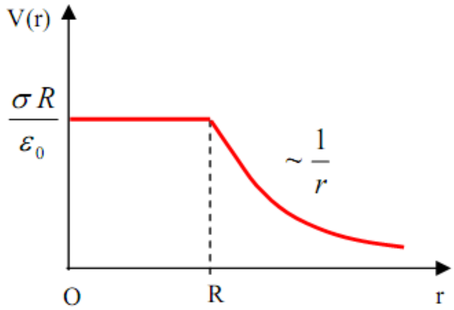
$V_1(R) = V_2(R)$

$$\begin{cases} V_2(R) = \frac{\sigma}{\varepsilon_0} R \\ V_1(R) = C_1 \end{cases} \Rightarrow C_1 = \frac{\sigma}{\varepsilon_0} R$$

$$\Rightarrow \boldsymbol{V_1 = \frac{\sigma}{\varepsilon_0} R}$$

**4) Graph/curve: $E(r)$ and $V(r)$.**



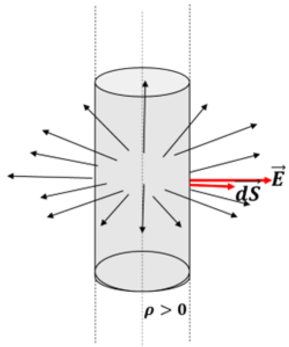

**Discussion**

The electrostatic field $E(r)$ when crossing the charged surface experiences a discontinuity equal to $\sigma/\varepsilon_0$, while the electrostatic potential $V(r)$ remains continuous.

## Exo-08:

The field has a cylindrical symmetry around the axis Oz, under any rotation the field remains the same. By reason of symmetry, the electrostatic field created by a sphere is:

- Radial $\vec{E} = E\,\vec{u_r}$
- Depends only on the radius $r$ : $E(r)$.




**1) The electric field at any point M in space.**

To determine the field $E$ at any point in space, we use Gauss' theorem,

$$\boldsymbol{\Phi = \oiint \vec{E} \cdot \overrightarrow{dS_G} = \frac{Q_{int}}{\varepsilon_0}}$$

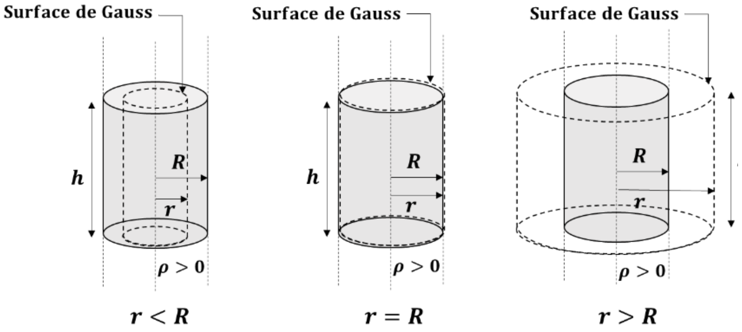
As the electrostatic field is radial, $\vec{E}$ and $\overrightarrow{dS}$ are collinear ($\vec{E} \parallel \overrightarrow{dS} \Rightarrow \theta = 0°$, we can then write:

$$\Rightarrow \oiint E.dS_G = \frac{Q_{int}}{\varepsilon_0}$$

$$\Rightarrow \boldsymbol{E.S_G = \frac{Q_{int}}{\varepsilon_0}} \ldots (*)$$

Due to cylindrical symmetry, the suitable closed Gauss surface $S_G$ here is a lateral surface of a cylinder of radius $r$ and height h: ($S_G = 2\pi r h$).

To determine the field at any point M in space, we distinguish 03-regions:




**The electric field in the cylinder $r < R$ :**

In this case a part of the charge ported by the cylinder is located inside the Gaussian surface. So:

$$Q = \iiint \rho dV = \rho V = \pi r^2 h\rho$$

$$\begin{cases} S_G = 2\pi r\, h \\ Q_{int} = \pi r^2 h\rho \end{cases} \Rightarrow l'eq(*) \Rightarrow E_1 2\pi r h = \frac{\pi r^2 h\, \rho}{\varepsilon_0}$$

$$\Rightarrow \boldsymbol{E_1} = \frac{\boldsymbol{\rho\, r}}{\boldsymbol{2\varepsilon_0}}$$

**The electric field on the surface of the cylinder $r = R$ :**

In this case the Gaussian surface contains all the electric charge of the cylinder.

$$\begin{cases} S_G = 2\pi r\, h \\ Q_{int} = \pi R^2 h\rho \end{cases} \Rightarrow l'eq(*) \Rightarrow E_2 2\pi r h = \frac{\pi R^2 h\, \rho}{\varepsilon_0}$$

$$r = R \Rightarrow \boldsymbol{E_2} = \frac{\boldsymbol{\rho\, R}}{\boldsymbol{2\varepsilon_0}}$$

**The electric field outside the cylinder $r > R$ :**

Likewise, the Gaussian surface contains all the electric charge of the cylinder.

$$\begin{cases} S_G = 2\pi r\, h \\ Q_{int} = \pi R^2 h\rho \end{cases} \Rightarrow l'eq(*) \Rightarrow E_3 2\pi r h = \frac{\pi R^2 h\, \rho}{\varepsilon_0}$$

$$\Rightarrow \boldsymbol{E_3} = \frac{\boldsymbol{\rho\, R^2}}{\boldsymbol{2\varepsilon_0\, r}}$$

**Note:**

We can obtain the expression of the field $E_2$ on the surface of the cylinder ($r = R$), by replacing with $R$ either in the expression of $E_1$ (inside the cylinder), or in the expression of $E_3$ (found outside the cylinder).

**2) Check that the electric field is continuous at the surface of the cylinder.**

in $r = R$ : $\boldsymbol{E_1(R) = E_2(R) = E_3(R)}$, the electric field is therefore continuous across the cylinder

**Exo-09:**

**1) The total charge $Q$ of the sphere.**

$$Q_T = \int dq = \iiint \rho dV = \rho V \rightarrow \boldsymbol{Q_T = \frac{4}{3}\pi R^3 \rho}$$

**2) The expression of the electric field $E(r)$ at any point $M$ in space.**

Gauss' theorem:

$$\boldsymbol{\phi = \oiint \vec{E}\,.\,\overrightarrow{dS_G} = \frac{Q_{int}}{\varepsilon_0}}$$

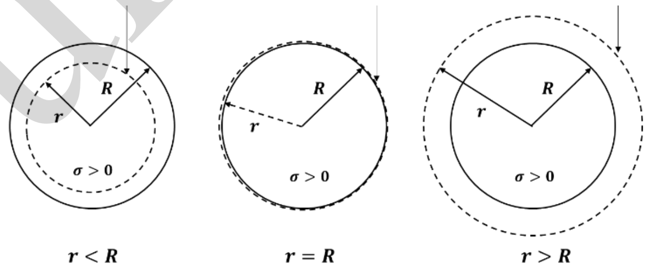
As the electrostatic field is radial, $\vec{E}$ and $\overrightarrow{dS}$ are collinear ($\vec{E} \parallel \overrightarrow{dS} \Rightarrow \theta = 0°$), we can then write:

$$\Rightarrow \oiint E.dS_G \cos\theta = \frac{Q_{int}}{\varepsilon_0}; (\vec{E} \parallel \overrightarrow{dS} \Rightarrow \theta = 0°)$$

$$\Rightarrow \boldsymbol{E.S_G = \frac{Q_{int}}{\varepsilon_0}} \ldots (*)$$

Due to spherical symmetry, the suitable closed Gauss surface $S_G$ here is a surface of sphere with center $O$ and radius $r$. ($S_G = 4\pi r^2$).

To determine the field at any point M in space, we distinguish 03-regions:




**Region-1 $r < R$ :**

$$\begin{cases} S_G = 4\pi r^2 \\ Q_{int} = \frac{4}{3}\pi r^3 \rho \end{cases} \Rightarrow l'eq(*) \Rightarrow \boldsymbol{E_1} = \frac{\boldsymbol{\rho}}{\boldsymbol{3\,\varepsilon_0}}\boldsymbol{r}$$

**Region-2 $r = R$ :**

$$\begin{cases} S_G = 4\pi r^2 \\ Q_{int} = \frac{4}{3}\pi R^3 \rho \end{cases} \Rightarrow l'eq(*) \Rightarrow \boldsymbol{E_2} = \frac{\boldsymbol{\rho}}{\boldsymbol{3\,\varepsilon_0}}\frac{\boldsymbol{R^3}}{\boldsymbol{r^2}}$$

$$r = R \Rightarrow \boldsymbol{E_2} = \frac{\boldsymbol{\rho R}}{\boldsymbol{3\,\varepsilon_0}}$$

**Region-3 r > R :**

$$\begin{cases} S_G = 4\pi r^2 \\ Q_{int} = \frac{4}{3}\pi R^3 \rho \end{cases} \Rightarrow l'eq(*) \Rightarrow E_3 = \frac{\rho}{3\,\varepsilon_0}\frac{R^3}{r^2} \Rightarrow \boldsymbol{E_3} = \frac{\boldsymbol{\rho}}{\boldsymbol{3\,\varepsilon_0}}\frac{\boldsymbol{R^3}}{\boldsymbol{r^2}}$$

**Remarks:**

We can obtain the expression of the field $\boldsymbol{E_2}$ on the surface of the sphere ($r = R$), by replacing $\boldsymbol{r}$ with $R$ in the expression of $\boldsymbol{E_3}$ found outside the sphere or in $\boldsymbol{E_1}$ found inside the sphere.

**3) The electric potential $V(r)$**

To determine the potential, we use the relation:

$$\vec{E} = -\overrightarrow{grad}\, V$$

E is radial $\Rightarrow E = -\dfrac{dV}{dr} \Rightarrow V = -\int E\, dr$

**Region-3 $r < R$ :**

$$V_3 = -\int E_3\, dr = V = -\int \frac{\rho}{3\,\varepsilon_0}\frac{R^3}{r^2}\, dr = \frac{\rho}{3\,\varepsilon_0}\frac{R^3}{r} + C_3$$

To determine $C_3$, we use the limit conditions of the potential: $\lim\limits_{r \to \infty} V = 0$

$$\Rightarrow \lim_{r \to \infty} V_3 = 0 + C_3 = 0 \Rightarrow \mathbf{C_3 = 0} \Rightarrow \boldsymbol{V_3 = \frac{\rho}{3\,\varepsilon_0}\frac{R^3}{r}}$$

**Region-2 $r = R$ :**

$$V_2 = -\int E_2\, dr = -\int \frac{\rho R}{3\,\varepsilon_0}\, dr = -\frac{\rho R}{3\,\varepsilon_0} r + C_2$$

To determine $C_2$ , we use the condition of continuity of the potential at the interface $(r = R)$ :

$$V_3(r = R) = V_2(r = R)$$

$$\begin{cases} V_2(R) = -\dfrac{\rho R^2}{3\,\varepsilon_0} + C_2 \\ V_3(R) = \dfrac{\rho R^2}{3\,\varepsilon_0} \end{cases} \Rightarrow C_2 = 2\frac{\rho R^2}{3\,\varepsilon_0}$$

$$\Rightarrow \boldsymbol{V_2 = \frac{\rho R^2}{3\,\varepsilon_0}}$$

**Region-1 r > R :**

$$V_1 = -\int E_1\, dr = -\int \frac{\rho}{3\,\varepsilon_0} r\, dr = -\frac{\rho}{3\,\varepsilon_0}\frac{r^2}{2} + C_2$$

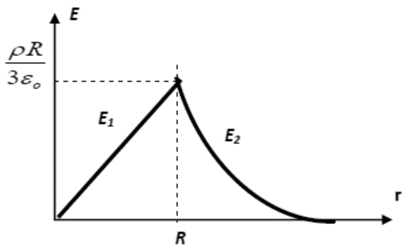
To determine $C_1$ , we use the condition of continuity of the potential at the interface $(r = R)$:

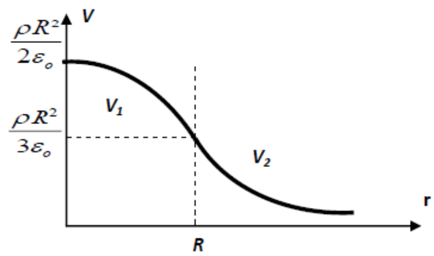
$$V_1(R) = V_2(R)$$

$$\begin{cases} V_2(R) = \dfrac{\rho R^2}{3\,\varepsilon_0} \\ V_1(R) = -\dfrac{\rho}{3\,\varepsilon_0}\dfrac{R^2}{2} + C_2 \end{cases} \Rightarrow C_1 = \frac{\rho R^2}{2\,\varepsilon_0}$$

$$\Rightarrow \boldsymbol{V_1 = \frac{\rho R^2}{3\,\varepsilon_0}\left[\frac{3}{2} - \frac{r^2}{2R^2}\right]}$$

**4) The variation curve of $E(r)$ and $V(r)$.**



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