Tutorial n°1: Electrostatics

Part-I: Discrete charge distribution

Exercise 1

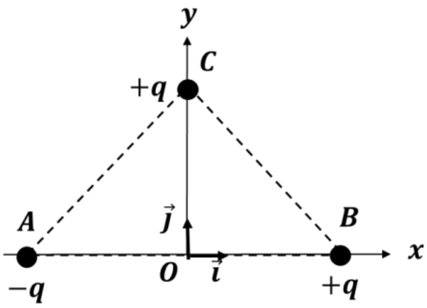
I. Calculate the electrostatic field (E) and the electrostatic potential (V) produced by an electric charge $q = 6 \times 10^{-10} C$ in point M at a distance d = 3 cm.

II. In a Cartesian reference frame, two electrical charges $q_A = 3 \times 10^{-7} C$ and $q_B = -10^{-7} C$ are placed respectively in points $A\ (1, 0, 2)$ and $B\ (3, -2, 4)$: (m). Determine the components of the electrostatic force and calculate its magnitude.

We give: $k = 9 \times 10^9\ N.m^2.C^{-2}$

Exercise 2

We consider three identical punctual charges $q_A = -q, q_B = +q\ and\ q_C = +q$ placed at the tops of a triangle, in $A\ (-R, 0)$, $B\ (R, 0)$ and $C\ (0, R)$, respectively.

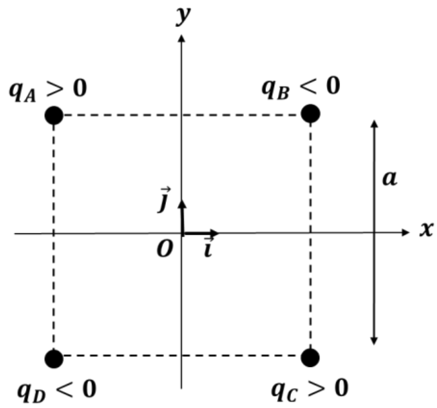


1- Give the expression of the electric field $\overrightarrow{E_O}$ at point O, and calculate its value.
2- Give the expression of electric potential V_O at point O, and calculate its value.
3- Calculate the electrostatic force acting on the charge $q_O = q/2$ placed in O.
4- Calculate the potential energy of the charge q_O.

We give: $k = 9 \times 10^9\ N.m^2.C^{-2}$; $q = 2 \times 10^{-9} C$; $R = 3\ cm$.

Exercise 3

We place four electric charges q_A, q_B, q_C and q_D at the tops ABCD of a square, with side a and center O, origin of an orthonormal base $(O, \vec{i}, \vec{j})$.



1- Give the expression of the electric field $\overrightarrow{E_O}$ at point O.
2- Give the expression of electric potential V_O at point O.

We give: $q_A = q,\ q_B = -2q,\ q_C = 2q\ et\ q_D = -q$.

Exercise 4 (Electric Flux)

Consider a point charge $+\boldsymbol{q}$ placed at point O on the OX axis. A circular disk of radius $\boldsymbol{R}$ is placed perpendicular to the OX axis at a distance $\boldsymbol{x}$ from point O.

Calculate the electrostatic flux $\boldsymbol{\varphi}$ of the electric field created by the charge $\boldsymbol{q}$ through the surface of the disk.

We give: $\int \frac{x\,dx}{\sqrt{x^2+y^2}^3} = \frac{-1}{\sqrt{x^2+y^2}}$

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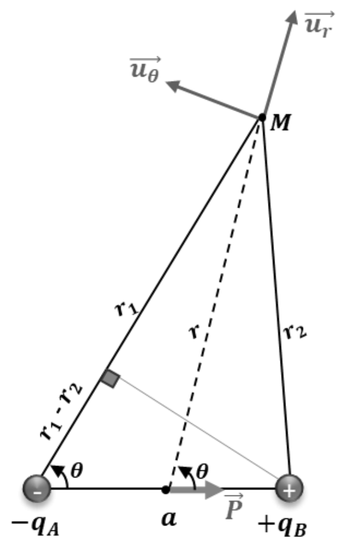
<u>Exercise 5</u> (Electric dipole)

An electric dipole consists of two-point charges of equal magnitude and opposite signs: ($q_A = -q$) and ($q_B = +q$) separated by a small distance $\boldsymbol{a}$. The dipole moment is defined by: $\vec{P} = q\,\overrightarrow{AB}$.



1- Determine the electric potential V_M created by the dipole at the point M, distant r from its center O, where ($r \gg a$).

2- Deduce the components E_r and E_θ of the electric field E at the point M, and its module $\|\vec{E}\|$ in the polar coordinate system.

3- For the following values of θ: $(\pi, \frac{\pi}{2})$, deduce the corresponding components E_r and E_θ.

4- We now assume that the dipole is subject to a uniform external field $\overrightarrow{E_{ext}}$.

- **a.** Give the expression of the electrostatic moment $\vec{L}$
- **b.** Determine the potential energy of the dipole E_p.

5- Graphically represent the electric field lines and equipotential surfaces of an electric dipole.

Exo- 1

1) The electric field and electric potential at M

$$\begin{cases} E = \dfrac{k|q|}{d^2} = 6.10^3\ V/m \\ V = \dfrac{kq}{d} = 180\ V \end{cases}$$

2) The electrostatic force exerted between q_A & q_B

We have:

$$\vec{F_e} = \frac{k\,|q_A q_B|}{r^3}\,\vec{r}$$

where: $\begin{cases} \vec{r} = \vec{AB} = 2\vec{i} - 2\vec{j} + 2\vec{k} \\ r = 2\sqrt{3} \end{cases}$

$$\Rightarrow \vec{F_e} = \left[-\frac{3\sqrt{3}}{4}\,\vec{i} + \frac{3\sqrt{3}}{4}\,\vec{j} - \frac{3\sqrt{3}}{4}\,\vec{k}\right] \times 10^{-5}$$

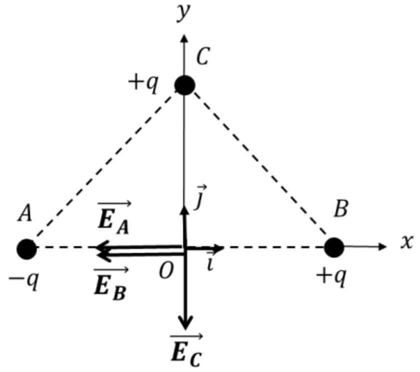
Magnitude : $F_e = \dfrac{9}{4} \times 10^{-5}\ N$

Exo- 2

1) The electric field $\vec{E_O}$

According to the principle of superposition:

$$\vec{E_O} = \sum_{i=1}^{3} \vec{E_i} = \vec{E_A} + \vec{E_B} + \vec{E_C}$$



$$\begin{cases} \vec{E_A} = -|E_A|\vec{i} \\ \vec{E_B} = -|E_B|\vec{i} \\ \vec{E_C} = -|E_C|\vec{i} \end{cases} \Rightarrow \begin{cases} \vec{E_A} = -\left|\dfrac{kq_A}{{r_A}^2}\right|\vec{i} \\ \vec{E_B} = -\left|\dfrac{kq_B}{{r_B}^2}\right|\vec{i} \\ \vec{E_C} = -\left|\dfrac{kq_C}{{r_C}^2}\right|\vec{j} \end{cases}$$

$$\Rightarrow \begin{cases} \vec{E_A} = -\dfrac{kq}{R^2}\vec{i} \\ \vec{E_B} = -\dfrac{kq}{R^2}\vec{i} \\ \vec{E_C} = -\dfrac{kq}{R^2}\vec{j} \end{cases} \Rightarrow \vec{E_O} = -\frac{kq}{R^2}(2\,\vec{i} + \vec{j})$$

Magnitude : $\|\vec{E_O}\| = -\frac{kq}{R^2}\sqrt{2^2 + 1^2}$

$$\Rightarrow E_O = \frac{\sqrt{5}\,kq}{R^2}$$

N.A: $E_O = 2\sqrt{5}\,.\,10^4\ V/m$

2) The electric potential V_O

According to the principle of superposition:

$$V_O = \sum_{i=1}^{3} V_i = V_A + V_B + V_C$$

$$V_O = \frac{kq_A}{r_A} + \frac{kq_B}{r_B} + \frac{kq_C}{r_C} \Rightarrow V_O = -\frac{kq}{R} + \frac{kq}{R} + \frac{kq}{R}$$

$$\Rightarrow V_O = \frac{kq}{R}$$

N.A: $V_O = 6\,.\,10^2\ V$

3) The electrostatic force F_O

$$\vec{F_O} = q_O\,.\,\vec{E_O} \Rightarrow F_O = q_O\,.\,E_O \Rightarrow F_O = \frac{\sqrt{5}\,kq^2}{2R^2}$$

N.A: $F_O = 2\sqrt{5}\,.\,10^{-5}\ N$

4) The potential energy E_p

$$E_P = q_O\,.\,V_O$$

$$\Rightarrow E_P = \frac{kq^2}{2R}$$

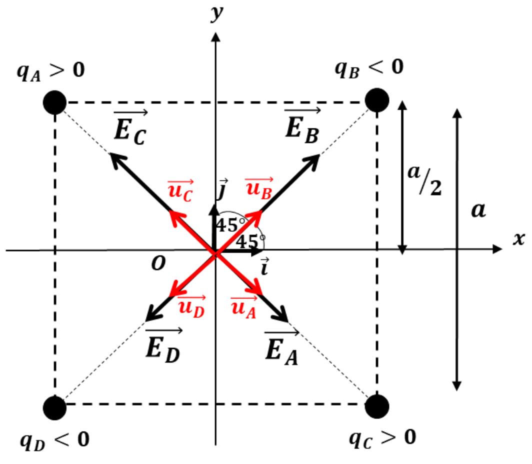
N.A: $E_P = 6\,.\,10^{-7}\ J$

Exo- 3

1) The electric field $\vec{E_O}$

According to the principle of superposition:

$$\vec{E_O} = \sum_{i=1}^{4} \vec{E_i} = \vec{E_A} + \vec{E_B} + \vec{E_C} + \vec{E_D}$$



and : $\vec{E_i} = |E_i|\,\vec{u_i} \Rightarrow \vec{E_i} = \left|\dfrac{kq_i}{{r_i}^2}\right|\vec{u_i}$

$$\begin{cases} \vec{E_A} = |E_A|\vec{u_A} \\ \vec{E_B} = |E_B|\vec{u_B} \\ \vec{E_C} = |E_C|\vec{u_C} \\ \vec{E_D} = |E_D|\vec{u_D} \end{cases} \Rightarrow \begin{cases} \vec{E_A} = \left|\dfrac{kq_A}{{r_A}^2}\right|\vec{u_A} \\ \vec{E_B} = \left|\dfrac{kq_B}{{r_B}^2}\right|\vec{u_B} \\ \vec{E_C} = \left|\dfrac{kq_C}{{r_C}^2}\right|\vec{u_C} \\ \vec{E_D} = \left|\dfrac{kq_D}{{r_D}^2}\right|\vec{u_D} \end{cases}$$

- **For the distances between the charges and O:**

  $r_A = r_B = r_C = r_D = \boldsymbol{r} = \frac{\sqrt{2}}{2}\boldsymbol{a}\,; \boldsymbol{r^2} = \frac{\boldsymbol{a}}{2}$
- **For unit vectors:**

  We have: $\vec{u} = \cos\alpha\ \vec{\imath} + \cos\beta\,\vec{\jmath} + \cos\gamma\ \vec{k}$

$$\begin{cases} \overrightarrow{u_A} = \cos 45°\ \vec{\imath} - \cos 45°\vec{\jmath} \\ \overrightarrow{u_B} = \cos 45°\ \vec{\imath} + \cos 45°\vec{\jmath} \\ \overrightarrow{u_C} = -\cos 45°\ \vec{\imath} + \cos 45°\vec{\jmath} \\ \overrightarrow{u_D} = -\ \cos 45°\ \vec{\imath} - \cos 45°\vec{\jmath} \end{cases} \begin{cases} \overrightarrow{u_A} = \frac{\sqrt{2}}{2}\ (\vec{\imath} - \vec{\jmath}) \\ \overrightarrow{u_B} = \frac{\sqrt{2}}{2}\ (\vec{\imath} + \vec{\jmath}) \\ \overrightarrow{u_C} = \frac{\sqrt{2}}{2}\ (-\vec{\imath} + \vec{\jmath}) \\ \overrightarrow{u_D} = \frac{\sqrt{2}}{2}\ (-\vec{\imath} - \vec{\jmath}) \end{cases}$$

$$\Rightarrow \begin{cases} \overrightarrow{E_A} = \frac{2kq}{a^2}\overrightarrow{u_A} \\ \overrightarrow{E_B} = \frac{4kq}{a^2}\overrightarrow{u_B} \\ \overrightarrow{E_C} = \frac{4kq}{a^2}\overrightarrow{u_C} \\ \overrightarrow{E_D} = \frac{2kq}{a^2}\overrightarrow{u_D} \end{cases} \Rightarrow \begin{cases} \overrightarrow{E_A} = \sqrt{2}\ \frac{kq}{a^2}(\vec{\imath} - \vec{\jmath}) \\ \overrightarrow{E_B} = 2\sqrt{2}\ \frac{kq}{a^2}\ (\vec{\imath} + \vec{\jmath}) \\ \overrightarrow{E_C} = 2\sqrt{2}\frac{kq}{a^2}\ (-\vec{\imath} + \vec{\jmath}) \\ \overrightarrow{E_D} = \sqrt{2}\frac{kq}{a^2}\ (-\vec{\imath} - \vec{\jmath}) \end{cases}$$

we calcul $\overrightarrow{\boldsymbol{E_O}}$

$$\Rightarrow \overrightarrow{E_O} = \sqrt{2}\ \frac{kq}{a^2}(\vec{\imath} - \vec{\jmath} + 2\vec{\imath} + 2\vec{\jmath} - 2\vec{\imath} + 2\vec{\jmath} - \vec{\imath} - \vec{\jmath})$$

$$\Rightarrow \overrightarrow{\boldsymbol{E_O}} = \boldsymbol{2\sqrt{2}\ \frac{kq}{a^2}\ \vec{\jmath}}$$

2) The electric potential V_O

According to the principle of superposition:

$$V_O = \sum_{i=1}^{4} V_i = V_A + V_B + V_C + V_D$$

$$V_O = \frac{kq_A}{r_A} + \frac{kq_B}{r_B} + \frac{kq_C}{r_C} + \frac{kq_D}{r_D}$$

$$\Rightarrow \boldsymbol{V_O = \frac{kq}{r} - \frac{2kq}{r} + \frac{2kq}{r} - \frac{kq}{r} = 0\ V}$$

Exo- 4

1) The electrostatic flux φ

We have:

$$\boldsymbol{d\varphi = \vec{E}.\overrightarrow{dS} = E\ dS\cos\alpha}$$

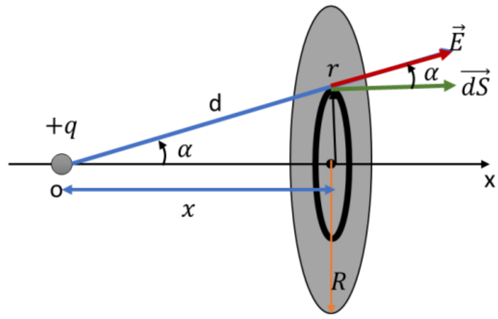
Knowing that:

$$E = \frac{kq}{d^2}\quad and\quad dS = 2\pi\, r\, dr$$

$$d^2 = r^2 + x^2\quad and\quad \cos\alpha = \frac{x}{d}$$

$$\varphi = \frac{q\,x}{2\,\varepsilon_0}\int_0^R \frac{r\,dr}{\sqrt{(r^2 + x^2)^3}}$$

$$\boldsymbol{\varphi = \frac{q}{2\,\varepsilon_0}\ (1 - \frac{x}{\sqrt{R^2 + x^2}})}$$



Exo- 5

1-The electric potential V_M created by the dipole

The electric potential V created by two charges q_A and q_B at point M is given by principle of superposition:

$$V_M = V_A + V_B$$

$$V_M = \frac{kq_A}{r_1} + \frac{kq_B}{r_2}$$

$$V_M = -\frac{kq}{r_1} + \frac{kq}{r_2} = kq\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

$$V_M = kq\left(\frac{r_1 - r_2}{r_1}\right)$$

As long as $r \gg a$,

we can consider: $\begin{cases} r_1 r_2 = r^2 \\ r_1 - r_2 = a\cos\theta \\ \vec{P} = q\,\vec{a} \end{cases}$

$$\Rightarrow \boldsymbol{V_M = \frac{k\,P\cos\theta}{r^2}}$$

2- The electric field E_M created by the dipole

We use the relation: $\vec{E} = -\overrightarrow{grad}\ V$ in the polar coordinate system:

$$\begin{cases} E_r = -\frac{\partial V}{\partial r} = \boldsymbol{\frac{2k\,P\cos\theta}{r^3}} \\ E_\theta = -\frac{1}{r}\frac{\partial V}{\partial\theta} = \boldsymbol{\frac{k\,P\sin\theta}{r^3}} \end{cases}$$

Magnitude $E_M = \sqrt{{E_r}^2 + {E_\theta}^2}$

$$\boldsymbol{E_M = \frac{k\,P}{r^3}\sqrt{1 + 3\ \cos^2\theta}}$$

3-Particular cases

$$\boldsymbol{if}:\ \theta = 0 \Rightarrow \begin{cases} E_r = \frac{2k\,P}{r^3} \\ E_\theta = 0 \end{cases}$$

$$\boldsymbol{if}: \theta = \frac{\pi}{2} \Rightarrow \begin{cases} E_r = 0 \\ E_\theta = \frac{k\,P}{r^3} \end{cases}$$

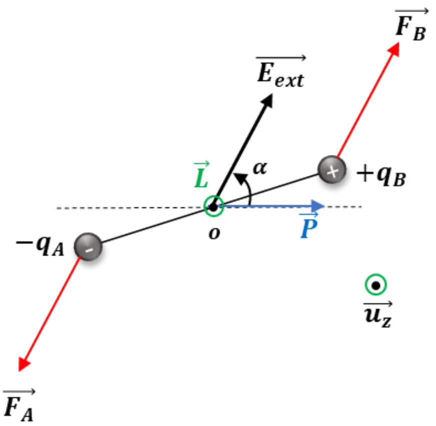
4- Electric dipole in an external electric field $\overrightarrow{E_{ext}}$

a- The electrostatic moment $\vec{L}$

When we place an electric dipole in an external field $\overrightarrow{E_{ext}}$, the dipole is subjected to electrostatic forces applied to its charges. These forces (equal and opposite) cause a couple moment $\vec{L}$ defined by:

$$\vec{L} = \vec{P} \wedge \overrightarrow{E_{ext}}$$

$$L = P\, E_{ext} \sin\alpha$$



b- Potential energy E_p of an electric dipole located in an external field

we have: $E_p = q\,V \;\Rightarrow\; dE_p = q\,dV \;\Rightarrow\; dE_p = q\left(-\overrightarrow{E_{ext}}\,.\,\overrightarrow{dl}\right) \;\Rightarrow E_p = -\,q\overrightarrow{E_{ext}}\,.\,\vec{a}$

$$\boldsymbol{E_p = -\,\overrightarrow{E_{ext}}\,.\,\vec{P}}$$

Particular cases:

$$\begin{cases} if\ \alpha = 0 & \Rightarrow E_p = -E_{ext}.P \quad \Rightarrow E_{p,min}(stable) \\ if\ \alpha = \pi & \Rightarrow E_p = +E_{ext}.P \quad \Rightarrow E_{p,max}(unstable) \end{cases}$$

5-Field lines and equipotential surfaces

Equipotentials are closed lines that surround the charge and are perpendicular to the field lines.

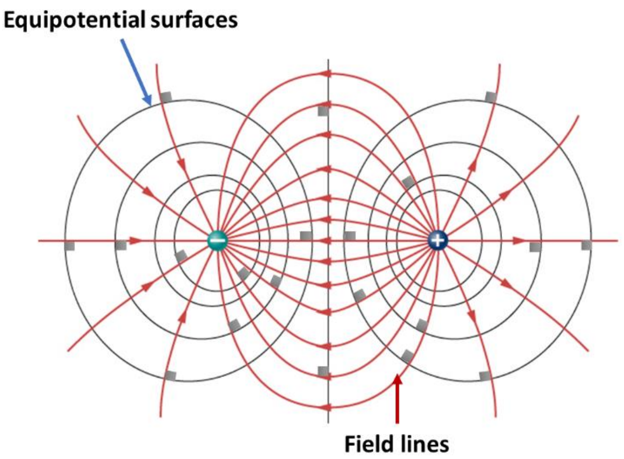


Figure: The field lines and equipotential lines of an electric dipole