# OEB University – L3 Mathematics Exam

Module: Optimization

January 10, 2026

**Duration:** 90 minutes
**Mark:** /20

**Exercise 1** (Theory)

1. Prove that if the function $f : \mathbb{R}^n \to \mathbb{R}$ is *strictly convex*, then the minimization problem
$$\inf_{x \in \mathbb{R}^n} f(x)$$
admits at most one solution.
2. Give an example of a function $f : \mathbb{R} \to \mathbb{R}$ which is *not coercive* but still admits a global minimum.
3. Give an example of a function for which the first-order condition is satisfied at a point that is not a local minimum.
4. What is the rate of convergence? How is it related to the order of convergence?
5. Define a descent direction. Give an example in $\mathbb{R}^2$.

**Exercise 2** (Critical Points)
Consider the function
$$f(x, y) = x^3 + 2xy - 2x^2 - 2y^2.$$

1. Find the critical points of $f(x, y)$.
2. Determine the nature of each critical point.
3. Are the extremums local or global? Justify your answer.
4. Are the extremums unique? Justify your answer.

**Exercise 3** (Gradient Method)
Consider the function
$$f(x, y) = 2x^2 + 3y^2 - 2xy + 5x - 6.$$

(a) Compute the gradient $\nabla f(x, y)$.

(b) Determine a **descent direction** at $x_0 = (1, 2)$.

(c) Using a fixed step size $\beta = 0.1$, compute the iterations $x_1$ and $x_2$ of the gradient method.