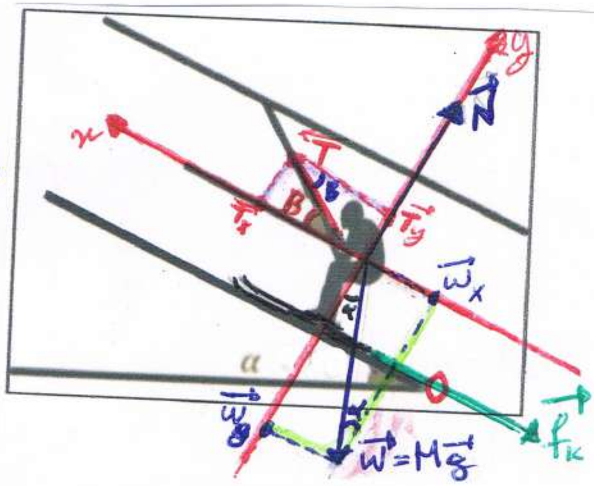
Series 4

Ex 1:

1) Represent the external forces exerted on the skier (see figure)



→ write their coordinates in the frame (Oxy):

$$\vec{w}\begin{cases} w_x = -w\sin\alpha \\ w_y = -w\cos\alpha \end{cases} \qquad \vec{N}\begin{cases} N_x = 0 \\ N_y = N \end{cases}$$

$$\vec{f_k}\begin{cases} f_{k_x} = -f_k \\ f_{k_y} = 0 \end{cases} \qquad \vec{T}\begin{cases} T_x = +T\cos B \\ T_y = +T\sin B \end{cases}$$

2) Prove that the skier is moving constant acceleration (Uniformly accelerated motion $\Rightarrow \vec{a} = C$)

→ Newton's second law application:

$$\sum \vec{F}_{ext} = M\vec{a} \qquad \underline{NB}\ \boxed{\vec{a} = a_x + \cancel{a_y}_{0} \Rightarrow \begin{cases} a_x = ?? \\ a_y = 0 \end{cases}}$$

$$\vec{w} + \vec{f_k} + \vec{N} + \vec{T} = M\vec{a}$$

the projection on an axis $(OX) \Rightarrow$

$$\Rightarrow -w\sin\alpha - f_k + 0 + T\cos B = M a_x$$

$$\Rightarrow a_x = \frac{-Mg\sin\alpha - f_k + T\cos B}{M}$$

$$a_x = \frac{-60(9,81)\sin 10 - 50 + 300\cos 25}{60}$$

$$\boxed{a_x = 1,99\ (m/s^2)} = C \Rightarrow$$

Uniformly accelerated motion

$$\Rightarrow \vec{a} = a_x = 1,99\ (m/s^2)$$

$$\Rightarrow |\vec{a}| = \sqrt{(1,99)^2} = 1,99\ m/s^2$$

3/ the time t ?? for the skier have travel $x = 400m$

we have: $a = \frac{dv}{dt} \Rightarrow dv = a\,dt \Rightarrow \int_{v_0}^{v} dv = \int_{t_0}^{t} 1.99 . dt$

$v - \cancel{v_0} = 1.99t$ with ($v_0 = 0 (m/s)$ at the beginning "0")

$\Rightarrow v = 1.99t$ (1)

we have: $v = \frac{dx}{dt} \Rightarrow dx = v.dt \Rightarrow \int_{x_0}^{x} dx = \int_{t_0}^{t} (1,99t).dt$

$\Rightarrow \overset{400m}{x} - \underset{0}{\cancel{x_0}} = \frac{1.99}{2} t^2$ ($x_0 = 0m$ at the beginning "0")

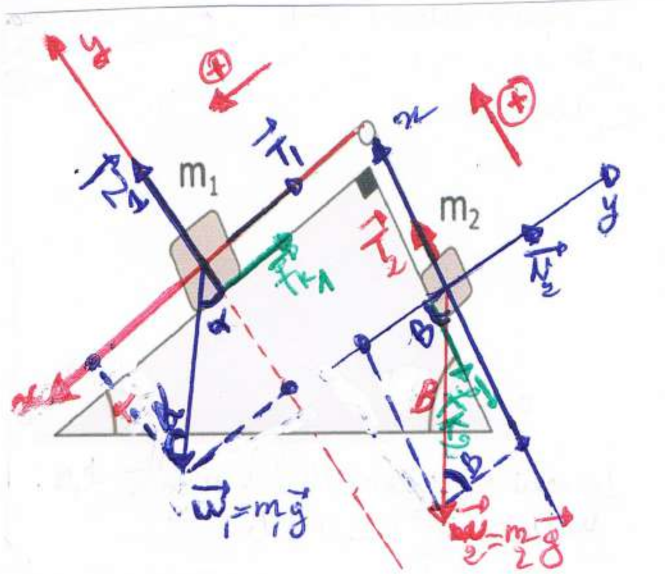
$\Rightarrow 400 = \frac{1.99}{2} t^2 \rightarrow t = \sqrt{\frac{2 \times 400}{1.99}} = 20,05\ s$

$\Rightarrow t = 20,05\ s$ (2)

→ the velocity at $t = 20{,}05\,s$

② → ① ⇒ $v = 1{,}99\,(20{,}05)$

$$v = 39{,}89\ (m/s^2)$$



EX2

1/ the intensity of the Normal force N_1 and N_2 for two masses (blocks).

$$\sum \vec{F}_{ext} = m\vec{a}$$

→ for the masse 1 : $m_1 = 9\,kg$

$$\sum \vec{F}_{ext} = m_1 \vec{a}$$

$$\vec{W}_1 + \vec{N}_1 + \vec{f}_{k1} + \vec{T}_1 = m_1 \vec{a}$$

* the projection on an axis (oy) ⇒

$$-W_1 \cos\alpha + N_1 + 0 + 0 = m_1 (0)$$

$$\Rightarrow N_1 = W_1 \cos\alpha \Rightarrow N_1 = \overbrace{m_1 g}^{W_1} \cos\alpha$$

$$\Rightarrow N_1 = 9\,(9{,}81)\cos 40 \Rightarrow N_1 = 67{,}63\,N \quad ①$$

→ for the masse 2 (m_2) :

$$\sum \vec{F}_{ext} = m_2 \vec{a} \Rightarrow \vec{W}_2 + \vec{N}_2 + \vec{f}_{k2} + \vec{T}_2 = m_2 \vec{a}$$

* the projection on an ascis (oy):

$\Rightarrow -W_2 \cos B + N_2 = 0 \Rightarrow N_2 = W_2 \cos B = m_2 g \cos B$

$\Rightarrow N_2 = 2(9,81) \cos 50 \Rightarrow N_2 = 12,61\, N$ ②

2/ the Kinetic forces f_{k_1} and f_{k_2}:

we have: $f_k = \mu_k \cdot N \rightarrow (*)$

① → (*):

$\Rightarrow$ for the masse (m_1): $f_{k_1} = \mu_k \cdot N_1$

$\Rightarrow f_{k_1} = 0,2 \times 67,63 \Rightarrow f_{k_1} = 13,52\, N$

② → (*) $\Rightarrow f_{k_2} = 0,2 \times 12,61 \Rightarrow f_{k_2} = 2,52\, N$

3/ the acceleration of the bloks:

$\sum \vec{F}_{ext} = m \vec{a}_x \Rightarrow \vec{W} + \vec{N} + \vec{f}_k + \vec{T} = m \vec{a}$

NB we have
$$\boxed{\begin{aligned} &T_1 = T_2 = T \text{ (Newton's third Low)} \\ &a_1 = a_2 = a \end{aligned}}$$

* for the masse 1 (block):

→ the projetion on an ascis (ox):

$$\overset{mg}{\widehat{W_1}} \sin \alpha - f_{k_1} - T = m_1 a$$

$$9(9{,}81) \sin 40 - 13{,}52 - T = 9a$$

$$\Rightarrow \boxed{43{,}23 - T = 9a} \quad (A)$$

→ for the masse ($m_2 = 2kg$): the projection (ox):

$$-W_2 \sin B - f_{k_2} + T = 2a$$

$$-2(9{,}81) \sin 50 - 2{,}52 + T = 2a$$

$$\Rightarrow \boxed{-17{,}54 + T = 2a} \quad (B)$$

$$(A) + (B) \Rightarrow 43{,}23 - \cancel{T} - 17{,}54 + \cancel{T} = 9a + 2a$$

$$25{,}68 = 11a$$

$$\Rightarrow a = 2{,}33 \ (m/s^2)$$