## 4.6 Homework

Exercise 4.13 Determine whether the series with general term

$$\sum_{n=1}^{\infty} a_n$$

converges, where

$$a_n = \begin{cases} \frac{1}{2^n}, & \text{if } n \text{ is even,} \\ \frac{1}{2^{n+1}}, & \text{if } n \text{ is odd.} \end{cases}$$

Exercise 4.14 Test the convergence of the series

$$\sum_{n=1}^{\infty} \frac{4^n \, n! \, n!}{(2n)!}$$

Exercise 4.15 Test the convergence of the series

$$\sum_{n=1}^{\infty} \frac{n!}{n^n}.$$

Exercise 4.16 Test the convergence of the series

$$\sum_{n=1}^{\infty} \frac{\sin n}{n}$$

Exercise 4.17 Determine the nature (convergence or divergence) of the following series:

1. 
$$u_n = \frac{n^3}{n!}$$
,

$$2. \ u_n = \frac{\ln(n^n)}{(\ln n)^n},$$

3. 
$$u_n = \frac{\sqrt[3]{n^4 + 1}}{n\sqrt{n - 1}}$$

4. 
$$u_n = \frac{1}{(\ln n)^p}$$
, with  $n > 1$ ,  $p > 0$ ,

Exercise 4.17 Determine the nature (converge)

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$$u_n = \frac{n^3}{n!}$$
,

2.  $u_n = \frac{\ln(n^n)}{(\ln n)^n}$ ,

3.  $u_n = \frac{\sqrt[3]{n^4 + 1}}{n\sqrt{n - 1}}$ ,

4.  $u_n = \frac{1}{(\ln n)^p}$ , with  $n > 1$ ,  $p > 0$ ,

5.  $u_n = \frac{(-1)^{n+1}}{\sqrt{n}}$ , with  $n \ge 1$ .