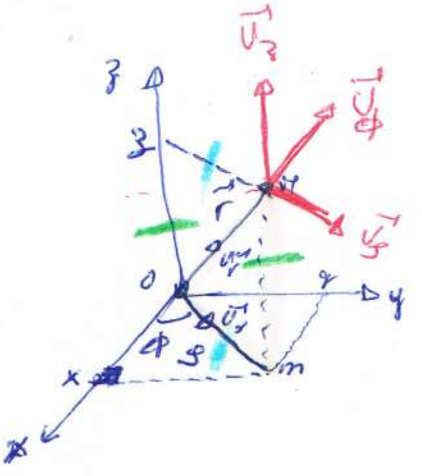
Series 3

Ex1 :



① the cylindrical coordinate (ρ, ϕ, z)

we have: $\rho = \sqrt{x^2+y^2}$ مثلث قائم في x لاننا أسقطنا (OxM)

$$\rho = \sqrt{(a e^{-kt}\cos kt)^2 + (a e^{-kt}\sin kt)^2}$$

$$\rho = \sqrt{(a e^{-kt})^2 \underbrace{(\cos^2 kt + \sin^2 kt)}_{1}}$$

a and k = C

$$\boxed{\rho = a e^{-kt}} \;(1) \Rightarrow \dot{\rho} = -ak e^{-kt} \;(2) \Rightarrow \ddot{\rho} = -ak^2 e^{-kt} \;(5)$$

$*\; tg\,\phi = \dfrac{y}{x}$ مقابل / المجاور $\Rightarrow tg\,\phi = \dfrac{\cancel{a e^{-kt}}\sin kt}{\cancel{a e^{-kt}}\cos kt} \Rightarrow tg\,\phi = \dfrac{\sin kt}{\cos kt}$ $tg\,kt$

$$\Rightarrow tg\,\phi = tg\,kt \Rightarrow \boxed{\phi = kt} + \pi k$$

$$\Rightarrow \underset{cte}{\dot{\phi} = k} \;(3) \Rightarrow \ddot{\phi} = 0 \;(6)$$

$*\; z_{cylin} = z_{carti} = t^3$

$$\Rightarrow \dot{z} = 3t^2 \;(4) \Rightarrow \ddot{z} = 6t \;(7)$$

→ the position vector $\vec{OM}$ (cylindrical coordinate)

$$\vec{OM} = \vec{r} = \vec{Om} + \vec{mM} = \rho \vec{U}_\rho + z \vec{U}_z \quad \text{with} \begin{cases} \rho = a e^{-kt} \\ z = t^3 \end{cases}$$

$$\Rightarrow \boxed{\vec{OM} = a e^{-kt}\,\vec{U}_\rho + t^3\,\vec{U}_z} \quad \text{with} \begin{cases} \vec{U}_\rho = \cos \underbrace{kt}_{\phi}\,\vec{i} + \sin \underbrace{kt}_{\phi}\,\vec{j} \\ \vec{U}_z = \vec{k} \text{ لأنهما متوازيان متساويان} \end{cases}$$

② - the cylindrical components $(V_r, V_\phi$ and $V_z)$ of the $\vec{V}$:

$$\boxed{\vec{V} = \dot{\rho}\,\vec{U}_\rho + \rho\,\dot{\phi}\,\vec{U}_\phi + \dot{z}\,\vec{U}_z} \;(*)$$

$(1, 2, 3, 4) \to (*)$

$$\vec{V} = -ak e^{-kt}\,\vec{U}_\rho + a e^{-kt}(k)\,\vec{U}_\phi + 3t^2\,\vec{U}_z$$

$$\boxed{\vec{V} = \underbrace{-ak e^{-kt}}_{v_r}\,\vec{U}_\rho + \underbrace{ak e^{-kt}}_{v_\phi}\,\vec{U}_\phi + \underbrace{3t^2}_{v_z}\,\vec{U}_z}$$

with
$$\boxed{\begin{aligned} U_\rho &= \cos \overset{\phi}{\overline{kt}}\,\vec{i} + \sin \overset{\phi}{\overline{kt}}\,\vec{j} \\ U_\phi &= -\sin kt\,\vec{i} + \cos kt\,\vec{j} \\ \vec{U}_z &= \vec{k} \end{aligned}}$$

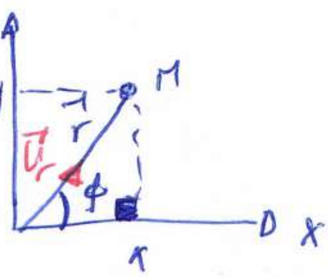
3- the cylindrical components (a_r, a_ϕ, a_z) of the $\vec{a}$

$$\boxed{\vec{a} = (\ddot{\rho} - \rho\dot{\phi}^2)\,\vec{U}_\rho + (2\dot{\rho}\dot{\phi} + \rho\ddot{\phi})\,\vec{U}_\phi + \ddot{z}\,\vec{U}_z}$$

$$\vec{a} = (\cancel{-ak^2e^{-kt}} - \cancel{ae^{-kt}k^2})\,\vec{U}_\rho + (2(-ake^{-kt})k + \cancel{ae^{-kt}(0)})\,\vec{U}_\phi + 6t\,\vec{U}_z$$

$$\boxed{\vec{a} = \underbrace{-2ak^2e^{-kt}}_{a_\phi}\,\vec{U}_\phi + \underbrace{6t}_{a_z}\,\vec{U}_z}$$

4- the position vector of point M in polar coordinates



$$\vec{OM} = r\,\vec{U}_r \longrightarrow (*)$$

$r = \sqrt{x^2 + y^2}$ من مثلث قائم في x نظرية فيتاغورث

$$r = \sqrt{(ae^{-kt}\cos kt)^2 + (ae^{-kt}\sin kt)^2}$$

$$\boxed{r = ae^{-kt}} \longrightarrow (1) \quad (2)\ \dot{r} = -kae^{-kt} \longrightarrow (4)\ \ddot{r} = +k^2ae^{-kt}$$

① → ✱ ⇒

$$\boxed{\vec{OM} = a e^{-kt} \vec{U}_r}$$

⑤ – the components of the Vector $\vec{V}$ (polar coordinate)

$$\boxed{\vec{V} = \dot{r} \vec{U}_r + r \dot{\phi} \vec{U}_\phi} \quad ✱$$

$$tg\,\phi = \frac{y}{x} = \frac{\cancel{a e^{-kt}} \sin kt}{\cancel{a e^{-kt}} \cos kt} \Rightarrow \quad tg\,kt$$

$$tg\,\phi = tg\,kt \Rightarrow \phi = kt + k\pi$$

$$\Rightarrow \dot{\phi} = k \;(3) \;\; \text{cte} \Rightarrow \ddot{\phi} = 0 \;(5)$$

$(1, 2, 3) \rightarrow (✱')$

$$\Rightarrow \boxed{\vec{V} = \underbrace{-k a e^{-kt} \vec{U}_r}_{v_r} + \underbrace{k a e^{-kt} \vec{U}_\phi}_{v_\phi}}$$

→ the components of the $\vec{a}$: (polar coordinate)

$$\vec{a} = (\ddot{r} - r\dot{\phi}^2)\vec{U}_r + (2\dot{r}\dot{\phi} + r\ddot{\phi})\vec{U}_\phi \quad (✱'')$$

① ② ③ ④, 5 → (✱'') !

$$\vec{a} = (+k^2 \cancel{a e^{-kt}} - k^2 \cancel{a e^{-kt}})\vec{U}_r + (-2k^2 a e^{-kt} + \cancel{a e^{-kt}(0)})\vec{U}_\phi$$

$$\boxed{\vec{a} = \underbrace{-2k^2 a e^{-kt}}_{a_\phi} \vec{U}_\phi}$$

Ex2 a Uniform circular motion

$$\begin{cases} v = ct \Rightarrow V = R\omega = C \\ a = a_N \Rightarrow a = a_N = R\omega^2 \end{cases}$$

(1) the angulair velocity ω:

$\omega = \frac{2\pi}{T}$ (1)

$T = 24\,h \Rightarrow T = 24 \times 3600 = 86\,400\,S \Rightarrow \boxed{T = 86400\,S}$ (2)

(2) → (1) : $W = \frac{2\pi}{86400} = 7{,}26 \cdot 10^{-5}\ (rad/s)$

(2)_ Acceleration (Frenet) :

$$\vec{a} = \underbrace{\cancel{\frac{dv}{dt}}}_{a_T} \vec{U}_T + \underbrace{\frac{v^2}{R=r}}_{a_N} \vec{U}_N$$

0

$V = cte \Rightarrow \vec{a} = \frac{v^2}{R=r}\vec{U}_N = \vec{a}_N$

$a = r\omega^2 \Rightarrow a = 42{,}164 \times 10^3 \times 10^3\ (7{,}26 \times 10^{-5})^2$

$a = 0{,}222\ (m/s^2)$

(3) the orbital velocity $\vec{V}$:

$a = \frac{v^2}{r} \Rightarrow v = \sqrt{a \cdot r} = \sqrt{0{,}222\,(42{,}164 \times 10^3 \times 10^3)}$

$\Rightarrow v = R\omega \Rightarrow v = 3061{,}10\ (m/s)$

Ex3

1/ determine its angular velocity $\omega_1 = \dot{\theta}$

We have : $\vec{V} = \dot{r}\vec{U_r} + r\dot{\theta}\vec{U_\theta} + r\dot{\phi}\sin\theta\,\vec{U_\phi}$ (*)

Expression of velocity in spherical coordinates

$R = r = cte \Rightarrow \dot{R} = \dot{r} = 0$ (1)

$\phi = cte \Rightarrow \dot{\phi} = 0$ (2)

(1) and (2) $\longrightarrow$ (*) ,

$\vec{V} = \underset{R}{\underset{\shortparallel}{r}}\,\underset{\omega_1}{\underset{\shortparallel}{\dot{\theta}}}\,\vec{U_\theta} \Rightarrow V = \sqrt{(r\dot{\theta})^2}$

$V = R\omega_1 \Rightarrow \omega_1 = \frac{V}{R} = \frac{\overset{V}{\frac{450\times10^3}{3600}}}{6370\times10^3}$

$\boxed{\omega_1 = 1{,}96\times10^{-5}\ (rad/s)}$

(2) angular $\omega_2 = \dot{\phi}$

$\alpha = 32°$ latitude خط العرض

we have: $\theta + \alpha = 90° \Rightarrow \theta = 90° - \alpha = 90° - 32° = 58°$

$\Rightarrow \boxed{\theta = 58°} \Rightarrow \boxed{\dot{\theta} = 0}$ ③

① and ③ $\rightarrow$ ④ $\Rightarrow$

$\vec{V} = \underset{R}{\underset{\shortparallel}{r}}\ \underset{\omega_2}{\underset{\shortparallel}{\dot{\phi}}} \sin\theta\ \vec{U_\phi} \Rightarrow V = \sqrt{(R\omega_2 \sin\theta)^2}$

$V = R\omega_2 \sin\theta \Rightarrow \omega_2 = \dfrac{V}{R\sin\theta} = \dfrac{450 \times 10^3}{3600 \times 6370 \times 10^3 \sin 58}$

$\boxed{\omega_2 = 2{,}31 \times 10^{-5}\ (rad/s)}$