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Faculty of Exact Sciences, Natural and Life Sciences

1st year LMD (2025-2026)

Series 3

Exercise 1

The time equations of motion of a material point M with relative to R $(O,\vec{\imath},\vec{\jmath},\vec{k})$ are given by:

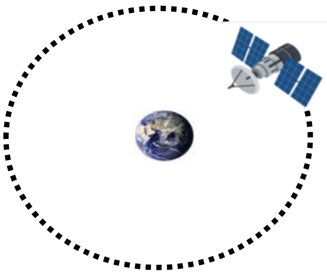
$$x(t) = ae^{-kt}\cos(kt), \quad y(t) = ae^{-kt}\sin(kt), \quad z(t) = t^3$$

In these equations a and k are constants.

1. Calculate the cylindrical coordinate (ρ, φ and z) of M as a function of t. Deduce the position vector of point M.
2. Determine the cylindrical components (V_ρ , V_φ and V_z) of the vector velocity $\vec{V}$.
3. Find the cylindrical components (a_ρ, a_φ and a_z) of the vector acceleration $\vec{a}$.
4. Write the position vector of point M in polar coordinates (we put z(t) =0).
5. Find the components of the vector velocity $\vec{V}$ and vector acceleration $\vec{a}$ in the polar coordinate.

Exercise 2

A geostationary satellite orbits the earth in a uniform circular motion with a radius of r (r =42,164 · 10^3km is the radius of the orbit from the center of the earth). The orbital period T for a geostationary satellite is approximately equal to 24h.



1. Find the angular velocity ω.
2. Determine the acceleration in the Frenet basis experienced by the geostationary satellite as it orbits the earth.
3. Deduce the orbital velocity of a geostationary satellite.

Exercise 3

We study the movement of an airplane flying at low altitude around the earth and we can model its trajectory using spherical coordinates. In this context, the earth can be thought of as a sphere with a radius R=6370km. The position of the airplane can be described using two angles θ and φ. Its velocity vector relative to the earth has a constant norm such that v=450Km/h.

1. The plane is flying moves from north to south, determine its angular velocity $\omega_1 = \dot{\theta}$.
2. Answer the same question if the planes moves from east to west on a circle whose latitude is α = 32° (we note that it's his angular velocity $\omega_2 = \dot{\varphi}$).