TD 04

Exercise 1

We equip $\mathbb{R}$ with the internal composition law defined as :

$$\forall x, y \in \mathbb{R}^+ : x \star y = \sqrt{x^2 + y^2}.$$

1. Show that $\star$ is commutative, associative, and has a neutral element.
2. Determine the symmetrizable elements.

Exercise 2

Show that $(G, \star)$ is a group and specify whether it is abelian (commutative) :

$$x \star y = \frac{x+y}{1+xy}, \text{ on } G = (-1, 1).$$

Exercise 3

Let be the set $\mathbb{R}^* \times \mathbb{R}$ provided with internal law $\star$ such that

$$(a, b) \star (\alpha, \beta) = (a\alpha, \frac{\beta}{a} + b\alpha)$$

Show that $(\mathbb{R}^* \times \mathbb{R}, \star)$ is a group, is it commutative ?

Exercise 4

Let $(G, +)$ be a commutative group. We denote $End(G)$ as the set of endomorphisms of G on which we define the operation $+$ as :

$$\begin{aligned} f + g : \quad G &\to G \\ x &\mapsto f(x) + g(x). \end{aligned}$$

Prove that $(End(G), +, \circ)$ is a ring.

Exercise 5

1. Determine if part H is a subgroup of group G.
   (a) $G = (\mathbb{Z}, +)$; $H = \{\text{even numbers}\}$
   (b) $G = (\mathbb{Z}, +)$; $H = \{\text{odd numbers}\}$.
2. Show that $U = \{z \in \mathbb{C}, |z| = 1\}$ equipped with multiplication is a subgroup of $(\mathbb{C}^*, \times)$.