TD 03

Exercise 1

Let P^* be the set of prime numbers strictly greater than 2.
We consider the relation $\mathcal{R}$ between two elements of P^* defined as :

$$p\mathcal{R}q \Leftrightarrow \frac{p+q}{2} \in P^*.$$

Is the relation $\mathcal{R}$ reflexive, symmetric, and transitive ?

Exercise 2

We define the relation $\mathcal{R}$ on $\mathbb{R}^2$ by :

$$(x,y)\mathcal{R}(x^{'},y^{'}) \Leftrightarrow \ x+y=x^{'}+y^{'}$$

1. Show that $\mathcal{R}$ is an equivalence relation.
2. Find the equivalence class of the couple $(0,0)$.

Exercise 3

Let R is a relation defined as

$$\forall x,y \in \mathbb{R},\ xRy \Leftrightarrow x^2-x=y^2-y,$$

1) Show that R is an equivalence relation.
2) Find the following equivalence classes : $\dot{0}$, $\dot{1}$, $\dot{2}$, $\dot{\frac{1}{2}}$.

Exercise 4

We define the relation $\mathcal{T}$ on $\mathbb{R}^2$ by

$$(x,y)\mathcal{T}(x^{'},y^{'}) \Leftrightarrow |x-x^{'}| \leq y^{'}-y.$$

1. Verify that $\mathcal{T}$ is an order relation. Is this order total ?
2. Let $(a,b) \in \mathbb{R}^2$ represent the set $\{(x,y) \in \mathbb{R}^2 / (x,y)\mathcal{T}(a,b)\}$.