

3.6 Homework

Exercise 3.10 For $t \in \mathbb{R}$, consider the following four differential equations:

(a) $y'(t) = -ty(t)$,

(b) $y'(t) = ty(t)$,

(c) $y'(t) = -t^2y(t)$,

(d) $y'(t) = -t^3y(t)$.

The graphs of the corresponding functions are shown in the figure 3.7. Without solving the differential equations, determine for each equation which of the given curves represents its solution.

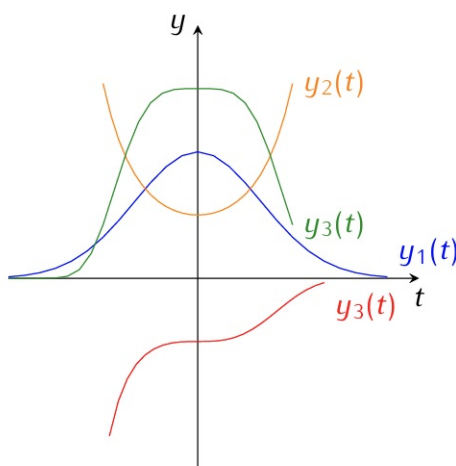


Figure 3.7

Exercise 3.11 Solve the equation

$$y' + \sin y + x \cos y + x = 0.$$

Exercise 3.12 Solve the differential equation

$$(y + xy^2)dx + xdy = 0.$$

Exercise 3.13 — Miscellaneous Problems. For each of Problems 1–32, solve the given differential equation. If an initial condition is provided, determine the particular solution that satisfies it.

1. $\frac{dy}{dx} = \frac{x^3 - 2y}{x}$

2. $(x + y)dx - (x - y)dy = 0$

3. $\frac{dy}{dx} = \frac{2x + y}{3 + 3y^2 - x}, \quad y(0) = 0$

4. $(x + e^y)dy - dx = 0$

5. $\frac{dy}{dx} = \frac{-2xy + y^2 + 1}{x + 2y}$
6. $x \frac{dy}{dx} + xy = 1 - y, \quad y(1) = 0$
7. $\frac{dy}{dx} = \frac{x^2y + y^3}{2xy + 1}, \quad \text{Hint: Let } u = x^2.$
8. $\frac{dy}{dx} = \frac{\sin x}{x}, \quad y(2) = 1$
9. $(3y^2 + 2xy)dx - (2xy + x^2)dy = 0$
10. $\frac{dy}{dx} + y = \frac{1}{1 + e^x}$
11. $(x^2 + y)dx + (x + e^y)dy = 0$
12. $\frac{dy}{dx} = (xy)^{1/2}$
13. $(x + 2y)dx - (x + y)dy = 0, \quad y(2) = 3$
14. $(x + y)dx + (x + 2y)dy = 0$
15. $(e^x + 1) \frac{dy}{dx} = y - ye^x$
16. $\frac{dy}{dx} = \frac{x^2 + y^2}{x^2}$
17. $\frac{dy}{dx} = e^{2x} + 3y$
18. $(2y + 3x)dx = -x dy$
19. $x dy - y dx = 2x^2y^2 dy, \quad y(1) = -2$
20. $y' = e^{x+y}$
21. $xy' = y + xe^{y/x}$
22. $\frac{dy}{dx} = \frac{x^2 - 1}{y^2 + 1}, \quad y(-1) = 1$
23. $xy' + y - y^2e^{2x} = 0$
24. $2 \sin y \cos x dx + \cos y \sin x dy = 0$
25. $\left(\frac{2x}{y} - \frac{y}{x^2 + y^2} \right) dx + \left(\frac{x}{x^2 + y^2} - \frac{x^2}{y^2} \right) dy = 0$
26. $(2y + 1)dx + \left(\frac{x^2 - y}{x} \right) dy = 0$
27. $(\cos 2y - \sin x)dx - 2 \tan x \sin 2y dy = 0$
28. $\frac{dy}{dx} = \frac{3x^2 - 2y - y^3}{2x + 3xy^2}$
29. $\frac{dy}{dx} = \frac{2y + \sqrt{x^2 - y^2}}{2x}$
30. $\frac{dy}{dx} = \frac{1 - 2xy^2}{x^2 + y^2 - 2x^2y^2}, \quad y(0) = 1$
31. $(x^2y + xy - y)dx + (x^2y - 2x^2)dy = 0$
32. $\frac{dy}{dx} = \frac{3x^2y + y^2}{2x^3 + 3xy}, \quad y(1) = -2$