2.5 Homework

Exercise 2.8 Show that the following integrals **converge**, and **compute their values**:

$$I_1 = \int_0^{+\infty} t^3 e^{-t}\, dt, \qquad I_2 = \int_1^{+\infty} \frac{1}{t\sqrt{t^2+1}}\, dt, \qquad I_3 = \int_0^{+\infty} \frac{t\ln(t)}{(t^2+1)^2}\, dt.$$

Exercise 2.9 Determine whether each of the following **improper integrals** is **convergent** or **divergent**:

$$I_1 = \int_2^{+\infty} \ln(t)\, dt, \qquad I_2 = \int_0^{2} \ln(t)\, dt, \qquad I_3 = \int_0^{+\infty} e^{-4t}\, dt,$$

$$I_4 = \int_0^{+\infty} e^{-t^2}\, dt, \qquad I_5 = \int_0^{+\infty} \frac{t^5}{(t^4+1)\sqrt{t}}\, dt, \qquad I_6 = \int_0^{\pi} \ln(\sin t)\, dt,$$

$$I_7 = \int_2^{+\infty} \left(1 - \cos\left(\tfrac{1}{t}\right)\right) dt, \qquad I_8 = \int_0^{1} \sin\left(\tfrac{1}{t}\right) dt, \qquad I_9 = \int_{\frac{2}{\pi}}^{+\infty} \ln\left(\cos\left(\tfrac{1}{t}\right)\right) dt.$$