Module: Algebra 01

First year

TD 02



Exercise 1

Let $A, B, C \in P(E)$, proving the following:

- 1. $A \setminus (B \cap C) = (A \setminus B) \cup (A \setminus C)$.
- 2. $A \subset B \Leftrightarrow A \cup B = B$
- 3. $A \cup B = A \cap C \Leftrightarrow B \subset A \subset C$



Exercise 2

Given A, B and C three parts of a set E,

- 1. Show that:
 - (a) $(A \cap B) \cup \mathcal{C}_E B = A \cup \mathcal{C}_E B$.
 - **(b)** $(A \setminus B) \setminus C = A \setminus (B \cup C)$.
- 2. Simplify:
 - (a) $\overline{(A \cup B)} \cap \overline{(C \cup \overline{A})}$.
 - **(b)** $\overline{(A \cap B)} \cup \overline{(C \cap \overline{A})}$.



Exercise 3

Let E, F two sets, $f: E \to F$ is an application, proving the following:

- 1. $\forall A, B \in P(E), f(A \cap B) \subset f(A) \cap f(B)$.
- 2. $\forall A, B \in P(E), f(A \cup B) = f(A) \cup f(B)$.
- 3. $\forall A \in P(F), f^{-1}(F \setminus A) = E \setminus f^{-1}(A).$



Exercise 4

- 1. Describe the direct image of $\mathbb R$ by the exponential function
- 2. Find f([0, 1]), $f(\mathbb{R})$, f(] 1, 2[), $f^{-1}(\{3\})$, par la fonction $f: x \mapsto x^2$, defined on \mathbb{R} .



Exercise 5

Let $f: [1; +\infty[\to [0; +\infty[$ is a function such that $f(x) = x^2 - 1$ bijective?



Exercise 6

Let E, F, G three sets, $f: E \to F, g: F \to G$ and $h: G \to E$

Proving that is $h \circ g \circ f$ is injective and if $g \circ f \circ h$ and $f \circ h \circ g$ are surjectives. Then, f, g and h are bijectives.