Series 2

EX1:

1/ the acceleration of this motion in each interval.

* **at [0-10s]:**

$$a = \frac{\Delta V}{\Delta t} = \frac{5-0}{10-0} = \frac{1}{2} = 0,5\ m/s^2 = Cst \Rightarrow$$

الحركة المستقيمة متغيرة بانتظام

* nature of the movement is: Uniformly Varied Retilinear motion OR rectilinear motion with uniformly accelerated

accelerated motion $V > 0$ and $a > 0$
$\vec{V} \cdot \vec{a} > 0$

* **at $t = [10-30s]$:** $V = 5\ m/s = Cst$

$\Rightarrow a = 0\ m/s^2$

الحركة المستقيمة منتظمة

* nature of the movement is: Unifom rectilinear motion

* **at $t = [30-40s]$:** $a = \frac{\Delta V}{\Delta t} = \frac{0-5}{40-30} = \frac{-5}{10} = -0,5\ m/s^2 <$

$a = Cst < 0 \Rightarrow$ Uniformly Varied rectilinear

الحركة المستقيمة متغيرة بانتظام

M^{vt} décéléré ≡ decelerated movement

EX2: movement 1D $\Rightarrow$ straight = rectilinear

$$a = 8t + 1 \ (m/s^2) \Rightarrow a = f(t).$$

1/ the Velocity of this movement at $t = 0$, $V = 2\ (m/s)$

$$a = \frac{dV}{dt} \Rightarrow dV = a \cdot dt$$

مراحل إيجاد V يجب أن نكامل
اشتقاق

$$\int_{V_0}^{V} dV = \int_{t_0}^{t} \underbrace{(8t+1)}_{a} dt \Rightarrow V\Big|_{V_0}^{V} = \frac{8t^2}{2} + t \Big|_{t_0=0}^{t}$$

$$V - V_0 = \frac{8t^2}{2} + t \Rightarrow V = \frac{8t^2}{2} + t + V_0 \ (1)$$

?? we have $\begin{cases} at = 0 \\ V = 2m/s \end{cases}$

$$\Rightarrow 2 = \frac{8(0)^2}{2} + 0 + V_0 \Rightarrow \boxed{V_0 = 2 (m/s)} \ (2)$$

$$(2) \rightarrow (1) \Rightarrow \boxed{V = \frac{8t^2}{2} + t + 2} \ (m/s) \ (*)$$

2/ the time equation x of this movement at $t = 0s$, $x = 0m$

$$V = \frac{dx}{dt} \Rightarrow dx = V dt \Rightarrow \int_{x_0}^{x} dx = \int_{t_0}^{t} \underbrace{\left(\frac{8t^2}{2} + t + 2\right)}_{V} dt$$

$$x\Big|_{x_0}^{x} = \frac{8t^3}{6} + \frac{t^2}{2} + 2t \Big|_{t_0=0}^{t}$$

$$x - x_0 = \frac{8t^3}{6} + \frac{t^2}{2} + 2t \Rightarrow x = \frac{8t^3}{6} + \frac{t^2}{2} + 2t + x_0$$

$t=0$, $x=0$

$$\Rightarrow 0 = 0 + x_0 \Rightarrow \boxed{x_0 = 0m}$$

a

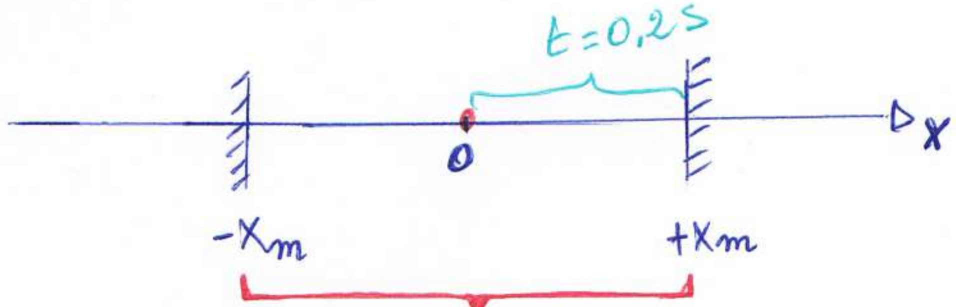
$$\boxed{x = \frac{8t^3}{6} + \frac{t^2}{2} + 2t}$$

3/ the nature of this movement is Rectilinear motion with variable acceleration because $a = f(t)$

Series 2

Ex3: $x(t) = X_m \cos(\omega t + \varphi)$

- Maximum elongation X_m??



$$2X_m = L = 6cm$$

we have $2X_m = L = 6cm \Rightarrow X_m = \frac{6}{2} = 3\ cm \Rightarrow \boxed{X_m = 3\ cm}$

- Period T:

we have: $T = 4t = 4(0,2) = 0,8s \Rightarrow \boxed{T = 0,8S}$

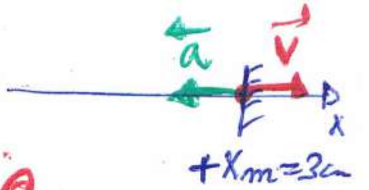
- Pulsation ω: $\omega = \frac{2\pi}{T} \Rightarrow \omega = \frac{2\pi}{0,8} \Rightarrow \boxed{\omega = 2,5\pi\ (rad/s)}$

- frequency f: $f = \frac{1}{T} \Rightarrow f = \frac{1}{0,8} \Rightarrow \boxed{f = 1.25\ Hz}$

- phase initiale φ: from the initial conditions at

$$t = 0 \begin{cases} x = 0 \\ V = 3(m/s) > 0 \end{cases}$$ mobile is at the origin

$x(t) = X_m \cos(\omega t + \varphi) \Rightarrow x(t) = \overset{cm}{3} \cos(2,5\pi t + \varphi)\ (cm)$



$$\Rightarrow \begin{cases} x(t) = \overset{cm}{3}\cos(2{,}5\pi t + \varphi) \ (cm) \\ V(t) = -\overset{cm}{3}(2{,}5\pi)\sin(2{,}5\pi t + \varphi) > 0 \end{cases}$$

$$\Rightarrow \text{at } t = 0 \Rightarrow \begin{cases} 0 = 3\cos(\cancel{2{,}5\pi(0)}^{0} + \varphi) \\ V = -3(2{,}5\pi)\sin(\cancel{2{,}5\pi(0)}^{0} + \varphi) > 0 \end{cases}$$

$$\Rightarrow \begin{cases} 3\cos\varphi = 0 \\ -7{,}5\pi\sin\varphi > 0 \end{cases} \Rightarrow \begin{cases} 3\cos\varphi = 0 \\ 7{,}5\pi\sin\varphi < 0 \end{cases} \Rightarrow$$

$\boxed{\varphi = \frac{-\pi}{2}}$ phase initiale

$$\Rightarrow \boxed{x(t) = 3\cos\left(2{,}5\pi t - \frac{\pi}{2}\right)}$$

2/ the $\underset{t??}{\underline{\text{time}}}$ does the mobile pass through the coordinate point $\underbrace{x = 3cm}_{X_m}$ for the first time is:

$$\text{at } t = t_1 \ \underset{X=3cm}{} \begin{cases} x = \cancel{\overset{cm}{3}}\cos\left(2{,}5\pi t_1 - \frac{\pi}{2}\right) = \cancel{3}\ cm \\ \underbrace{-7{,}5\pi\sin\left(2{,}5\pi t_1 - \frac{\pi}{2}\right)}_{V} > 0 \end{cases} \quad (V > 0)$$

$$\Rightarrow \begin{cases} \cos\left(2{,}5\pi t_1 - \frac{\pi}{2}\right) = 1 \\ \sin\left(2{,}5\pi t_1 - \frac{\pi}{2}\right) < 0 \end{cases} \Rightarrow 2{,}5\cancel{\pi} t_1 - \frac{\cancel{\pi}}{2} = 0 + 2k\cancel{\pi}$$

$$\Rightarrow 2,5\cancel{\pi}\, t_1 - \frac{\pi}{2} = 0 + 2k\cancel{\pi}$$

for $k=0 \Rightarrow 2,5\, t_1 = \frac{1}{2} + 2\cancel{k}^{\,0}$

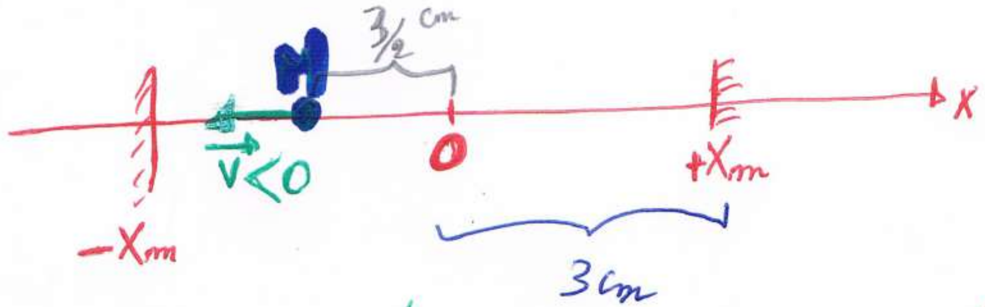
for $k=0 \Rightarrow 2,5\, t_1 = \frac{1}{2} \Rightarrow \boxed{t_1 = 0,2\,s}$

$\rightarrow$ Velocity at this date ($t_1 = 0,2s$) ant $x = X_m = 3cm$

$$V = -\overbrace{3\times10^{-2}}^{m}\,(2,5\pi)\sin\big(\underbrace{2,5\pi(0,2) - \frac{\pi}{2}}_{0}\big)$$

$\boxed{V = 0\,(m/s)}$ at $t_1 = 0,2s$ and $x = X_m = 3\,cm$

3/ the time (t_2?) the moving mobile passes through the coordinate point $x = \frac{3}{2}$ cm for the first time in the negative direction ($V<0$)



at $t = t_2$, $x = \frac{3}{2}$ cm $\Rightarrow \begin{cases} x(t) = \cancel{3}\cos\left(2,5\pi t_2 - \frac{\pi}{2}\right) = \frac{\cancel{3}}{2} \\ V(t) = -7,5\pi\sin\left(2,5\pi t_2 - \frac{\pi}{2}\right) < 0 \end{cases}$

$$\Rightarrow \begin{cases} \cos\left(2,5\pi t_2 - \frac{\pi}{2}\right) = \frac{1}{2} \\ \sin\left(2,5\pi t_2 - \frac{\pi}{2}\right) > 0 \end{cases}$$

$$\Rightarrow \begin{cases} \cos\left(2{,}5\pi t_2 - \frac{\pi}{2}\right) = \frac{1}{2} \\ \sin\left(2{,}5\pi t_2 - \frac{\pi}{2}\right) > 0 \end{cases}$$

$$\Rightarrow 2{,}5\cancel{\pi} t_2 - \frac{\cancel{\pi}}{2} = \frac{\cancel{\pi}}{3} + 2K\cancel{\pi}$$

$$\text{for } k=0 \Rightarrow 2{,}5\, t_2 = +\frac{1}{2} + \frac{1}{3} + \cancel{2K} \to 0$$

$$\underline{\text{for } k=0} \Rightarrow 2{,}5\, t_2 = \frac{3+2}{6} = \frac{5}{6}$$

$$\Rightarrow \boxed{t_2 = 0{,}33\,s}$$

$$\to (0, \tfrac{T}{4}, \tfrac{T}{2}, \tfrac{3T}{4}, T \ldots \tfrac{5T}{4} \ldots)$$

$$\begin{cases} x(t) = 3 \cdot 10^{-2} \cos\left(\underbrace{2{,}5\pi}_{\omega = \frac{2\pi}{T}} t - \frac{\pi}{2}\right) \; [m] \\ V(t) = -3 \times 10^{-2} (2{,}5\pi) \sin\left(2{,}5\pi t - \frac{\pi}{2}\right) \; [m/s] \\ a(t) = -3 \times 10^{-2} (2{,}5\pi)^2 \cos\left(2{,}5\pi t - \frac{\pi}{2}\right) \; [m/s^2] \end{cases}$$

$t(s)$	0	$T/4$	$T/2$	$\frac{3T}{4}$	T	$\frac{5T}{4}$ تعاود الحركة
$\omega t + \varphi_0 = \frac{2\pi}{T} t - \frac{\pi}{2}$	$-\frac{\pi}{2}$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	
$x(t) = 3\cos(\omega t + \varphi_0)$ cm	0	$+3$	0	-3	0	
$v(t) = -7{,}5\pi \sin(\omega t + \varphi_0)$ cm	$+7{,}5\pi$	0	$-7{,}5\pi$	0	$+7{,}5\pi$	
$a(t) = -18{,}75\pi^2 \cos(\omega t + \varphi_0)$	0	$-18{,}75\pi^2$	0	$+18{,}75\pi^2$	0