Serie 01

Ex 1

1) the equation of the trajectory of point M.

$$\begin{cases} x(t) = 2 + 3\cos\left(\frac{\pi}{2}t\right) \rightarrow ① \\ y(t) = 4 + 3\sin\left(\frac{\pi}{2}t\right) \rightarrow ② \end{cases}$$

$z(t) = 0$

$\varphi = wt + \varphi_0$
$\alpha = wt$
$\alpha = 30\pi t + \varphi_0$
$w = cte$

$$\Rightarrow \begin{cases} x - 2 = 3\cos\left(\frac{\pi}{2}t\right) \\ y - 4 = 3\sin\left(\frac{\pi}{2}t\right) \end{cases} \Rightarrow \begin{cases} (x-2)^2 = 3^2\cos^2\left(\frac{\pi}{2}t\right) ① \\ (y-4)^2 = 3^2\sin^2\left(\frac{\pi}{2}t\right) ② \end{cases}$$

$$① + ② \Rightarrow (x-2)^2 + (y-4)^2 = 3^2\Big[\underbrace{\cos^2\left(\tfrac{\pi}{2}t\right) + \sin^2\left(\tfrac{\pi}{2}t\right)}_{1}\Big]$$

$$\boxed{(x-2)^2 + (y-4)^2 = 3^2}$$

the trajectory of a circle with the centre is $(2,4)$ and radius $R = 3$

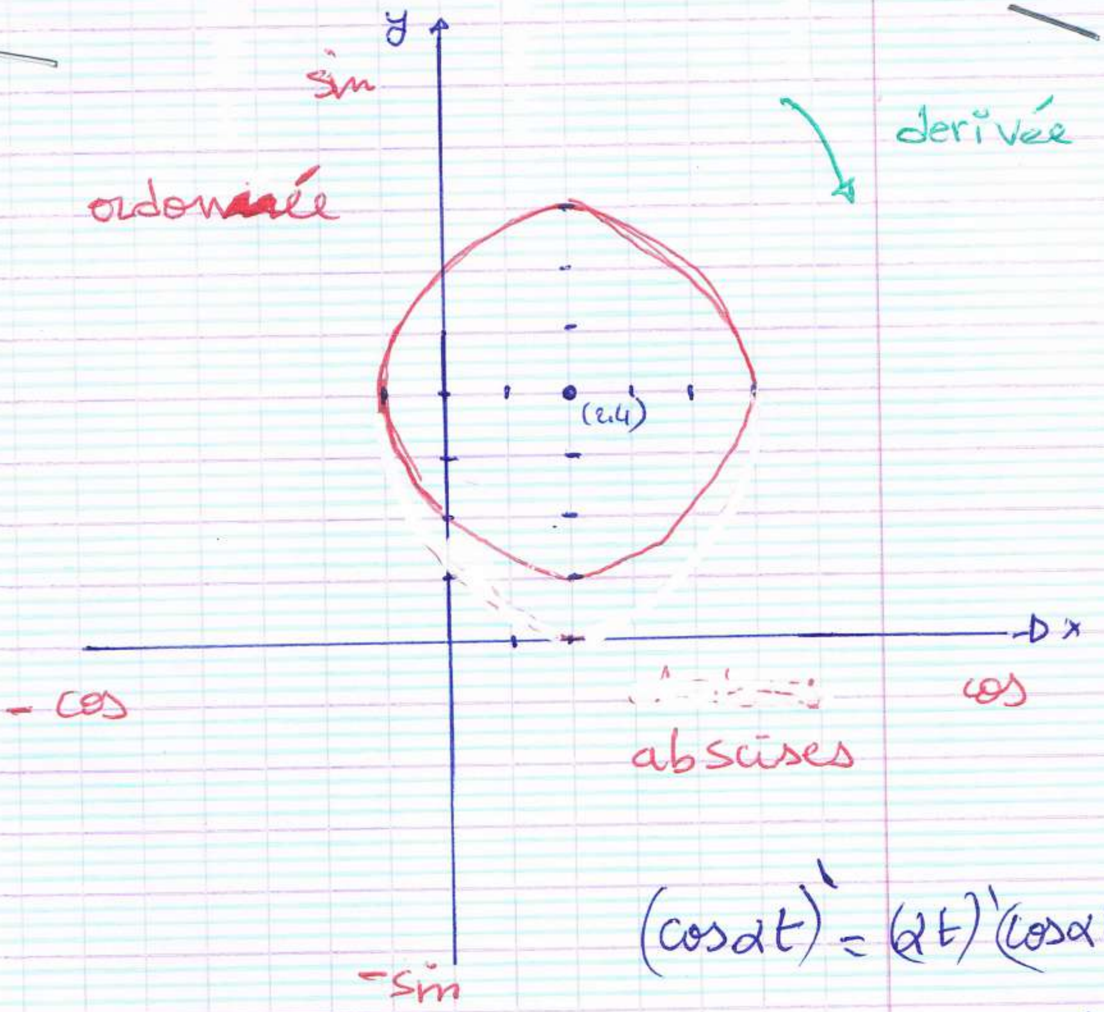
NB equation of a circle
equation d'un cercle: $(x - \underset{x_0}{a})^2 + (y - \underset{y_0}{b})^2 = R^2$

(a, b) centre de cercle

2)

$x(t) = 2 + 3\cos\left(\frac{\pi}{2}t\right)$ (degrée)	5	2	-1
$y(t) = 4 + 3\sin\left(\frac{\pi}{2}t\right)$	4	7	4
t	0	1	2

$$\boxed{\begin{aligned} \cos'\alpha &= -\sin\alpha \\ (\sin\alpha)' &= \cos\alpha \end{aligned}}$$



$$(\cos\alpha t)' = (\alpha t)'(\cos\alpha t)'$$
$$= \alpha(-\sin\alpha t)$$

3/ $\vec{OM} = \left[2+3\cos\left(\frac{\pi}{2}t\right)\right]\vec{i} + \left[4+3\sin\left(\frac{\pi}{2}t\right)\right]\vec{j}$ $\quad (\cos\alpha t)' = -(\cancel{\alpha t})'\sin$

4/ $\vec{V} = \frac{d\vec{OM}}{dt} = 3\left(-\frac{\pi}{2}\sin\left(\frac{\pi}{2}t\right)\right)\vec{i} + 3\left(\frac{\pi}{2}\cos\left(\frac{\pi}{2}t\right)\right)\vec{j}$

$$\vec{V} = \frac{d\vec{OM}}{dt} = \left[-\frac{3\pi}{2}\sin\left(\frac{\pi}{2}t\right)\right]\vec{i} + \left[\frac{3\pi}{2}\cos\left(\frac{\pi}{2}t\right)\right]\vec{j}$$

$$\vec{a} = \frac{d\vec{V}}{dt} = \left[-\frac{3\pi}{2}\left(\frac{\pi}{2}\right)\cos\left(\frac{\pi}{2}t\right)\right]\vec{i} + \left[-\left(\frac{3\pi}{2}\right)\left(\frac{\pi}{2}\right)\sin\left(\frac{\pi}{2}t\right)\right]\vec{j}$$

$$\vec{a} = \left[-\frac{3\pi^2}{4}\cos\left(\frac{\pi}{2}t\right)\right]\vec{i} - \left[\frac{3\pi^2}{4}\sin\left(\frac{\pi}{2}t\right)\right]\vec{j}$$

$$\begin{cases} x(t) = 2\cos\left(\frac{\pi}{2}t\right) \\ y(t) = 4\sin\left(\frac{\pi}{2}t\right) \\ z(t) = 0 \end{cases}$$

$$\begin{cases} x^2(t) = 2^2\cos^2\left(\frac{\pi}{2}t\right) \\ y^2 = 4^2\sin^2\left(\frac{\pi}{2}t\right) \end{cases} \Rightarrow \begin{cases} \frac{x^2}{2^2} = \cos^2\left(\frac{\pi}{2}t\right) \quad ① \\ \frac{y^2}{4^2} = \sin^2\left(\frac{\pi}{2}t\right) \quad ② \end{cases}$$

$① + ② \Rightarrow$

$$\boxed{\frac{x^2}{2^2} + \frac{y^2}{4^2} = 1}$$ ellipse trajectory

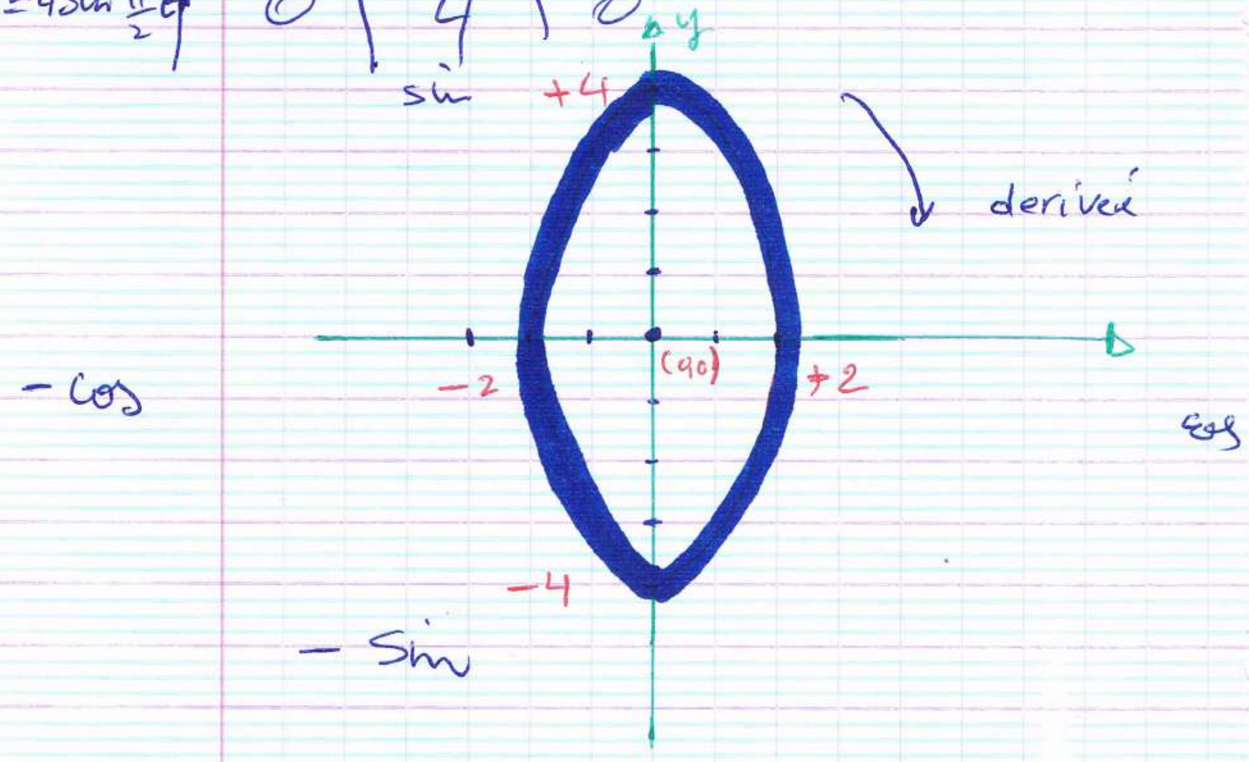
NB $$\boxed{\frac{(x-x_0)^2}{a^2} + \frac{(y-y_0)^2}{b^2} = 1}$$

$$\frac{(x-0)^2}{2^2} + \frac{(y-0)^2}{4^2} = 1$$ centre $(0,0)$

$(-2, +2)$ فواصل

$(-4, 4)$ ترتيب نرسم ثم منحنى

t	0	1	2
$x = 2\cos\frac{\pi}{2}t$	2	0	-2
$y = 4\sin\frac{\pi}{2}t$	0	4	0



3/ the expression of the position vector $\vec{OM}$:

$$\boxed{\vec{OM} = \left[2\cos\left(\frac{\pi}{2}t\right)\right]\vec{i} + \left[4\sin\left(\frac{\pi}{2}t\right)\right]\vec{j}}$$

4/ the velocity vector $\vec{V}$ and the acceleration vector $\vec{a}$

$$\vec{V} = \frac{d\vec{OM}}{dt} = \left[-\cancel{2}\,\frac{\pi}{\cancel{2}}\sin\left(\frac{\pi}{2}t\right)\right]\vec{i} + \left[\cancel{4}\left(\frac{\pi}{\cancel{2}}\right)\cos\left(\frac{\pi}{2}t\right)\right]\vec{j}$$

$$\boxed{\vec{V} = \left[-\pi\sin\left(\frac{\pi}{2}t\right)\right]\vec{i} + \left[2\pi\cos\left(\frac{\pi}{2}t\right)\right]\vec{j}}$$

$$\vec{a} = \frac{d\vec{V}}{dt} = \left[-\pi\left(\frac{\pi}{2}\right)\cos\left(\frac{\pi}{2}t\right)\right]\vec{i} + \left[-\cancel{2}\pi\left(\frac{\pi}{\cancel{2}}\right)\sin\left(\frac{\pi}{2}t\right)\right]\vec{j}$$

$$\boxed{\vec{a} = \left[-\frac{\pi^2}{2}\cos\left(\frac{\pi}{2}t\right)\right]\vec{i} + \left[\pi^2\sin\frac{\pi}{2}t\right]\vec{j}}$$

Ex2: $x = 3t + 2$, $y(t) = -t^2$, $z(t) = 3$

We have: $\vec{OM} = (3t+2)\vec{i} - t^2\vec{j} + 3\vec{k} \rightarrow (*)$

1/ the expression of the position vector $\vec{OM_1}$ at time $t = 1s$

$(*) \Rightarrow \vec{OM_1}(t = 1s) = (3(1)+2)\vec{i} - (1)^2\vec{j} + 3\vec{k}$

$\boxed{\vec{OM_1} = 5\vec{i} - \vec{j} + 3\vec{k}}$ at $t = 1s$. (1)

$(*) \Rightarrow \vec{OM_2}(t=2) = (3(2)+2)\vec{i} - (2)^2\vec{j} + 3\vec{k}$

$\boxed{\vec{OM_2} = 8\vec{i} - 4\vec{j} + 3\vec{k}}$ at $= t = 2s$ (2)

2/ the coordinates of the position variation vector $d\vec{OM}$

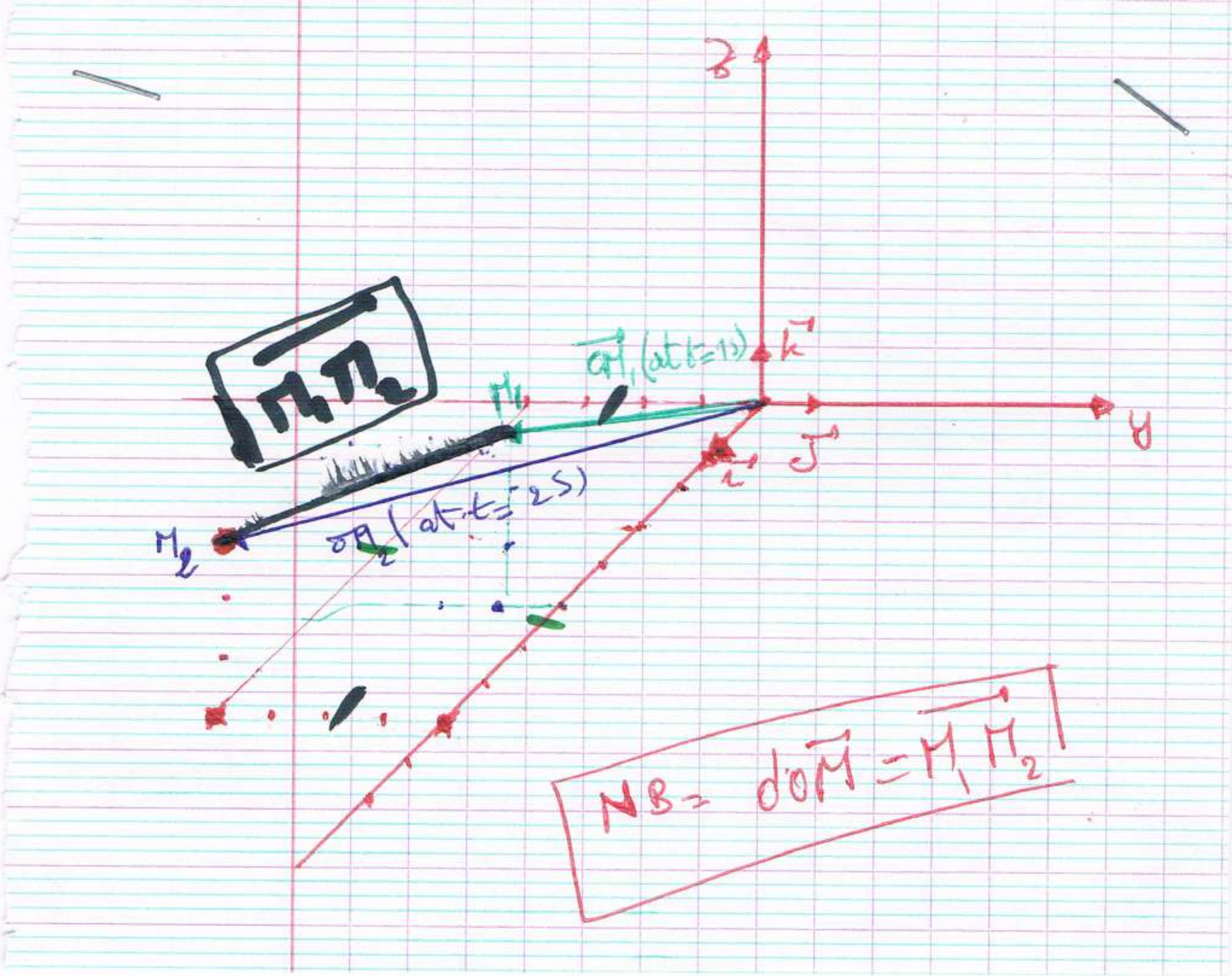
$d\vec{OM} = \vec{OM_2} - \vec{OM_1}$ with $dt = t_2 - t_1$ $(*')$

(1) and (2) $\rightarrow (*') \Rightarrow$

$d\vec{OM} = (8\vec{i} - 4\vec{j} + 3\vec{k}) - (5\vec{i} - \vec{j} + 3\vec{k})$

$\boxed{d\vec{OM} = 3\vec{i} - 3\vec{j} + 0\vec{k}}$

NB $d\vec{OM} = \vec{M_1M_2}$



Ex3 :

1/ Determine the trajectory of the mobile M $\boxed{y = f(x)}$

$$\begin{cases} x = \ln t \to (1) \\ y = t + \frac{1}{t} \to (2) \end{cases} \Longrightarrow \begin{cases} e^x = t \\ y = t + \frac{1}{t} \end{cases} \Longrightarrow \begin{cases} t = e^x & (1) \\ y = t + \frac{1}{t} & (2) \end{cases} \;??$$

$$(1) \to (2) \Longrightarrow y = e^x + \frac{1}{e^x} \Longrightarrow \boxed{y = e^x + e^{-x}}$$

2/ the Velocity Vector $\vec{V}$

we have; $\overrightarrow{OM} = (\ln t)\,\vec{i} + \left(t + \underset{}{\overset{= t^{-1}}{\frac{1}{t}}}\right)\vec{j}$

$$\left(t^{-n}\right)' = -n\,t^{-n-1}$$

$$\boxed{\vec{V} = \frac{d\overrightarrow{OM}}{dt} = \underbrace{\left(\frac{1}{t}\right)}_{V_x}\vec{i} + \underbrace{\left(1 - t^{-2}\right)}_{V_y}\vec{j}} \Longleftarrow \left(\frac{1}{t}\right)\vec{i} + \left(1 - \frac{1}{t^2}\right)\vec{j}$$

$$\left(t^{-1}\right)' = -1\,t^{-1-1}$$

→ Magnitude of $\vec{V}$: $V = \sqrt{V_x^2 + V_y^2}$

$$V = \sqrt{\left(\frac{1}{t}\right)^2 + \left(1 - t^{-2}\right)^2}$$

for $t = 1s \Rightarrow V = \sqrt{\left(\frac{1}{1}\right)^2 + \left(1 - \cancel{(1)^{-2}}\right)^2}$ $\leftarrow 0$

$$\boxed{V = 1\, m/s}$$

* <u>acceleration vector $\vec{a}$:</u>

$$\boxed{\vec{a} = \frac{d\vec{V}}{dt} = \underbrace{\left(-t^{-2}\right)}_{a_x}\vec{i} + \underbrace{\left(2t^{-3}\right)}_{a_y}\vec{J}} \Leftrightarrow \frac{-1}{t^2}\vec{i} + \frac{2}{t^3}\vec{J}$$

$$\Rightarrow a = \sqrt{\left(-t^{-2}\right)^2 + \left(2t^{-3}\right)^2} = \sqrt{\left(-\frac{1}{t^2}\right)^2 + \left(\frac{2}{t^3}\right)^2}$$

$$a = \sqrt{\left(\frac{-1}{(1)^2}\right)^2 + \left(\frac{2}{(1)^3}\right)^2} = \sqrt{1+4} = \sqrt{5}\, m/s^2$$