TP Session – Chapter 5
Gradient and Newton Methods

Mathematics Department

Objectives

- Understand iterative methods for unconstrained optimization.
- Implement and compare Gradient Descent and Newton's Method.
- Visualize convergence behavior in MATLAB.

Exercises

1. Consider the function $f(x) = (x-2)^2$.
   (a) Implement the **Gradient Descent** algorithm.
   (b) Test different learning rates α.
2. Consider the function $f(x) = x^4 - 3x^2 + 2$.
   (a) Derive the gradient and Hessian analytically.
   (b) Implement **Newton's Method** and compare convergence speed with Gradient Descent.
3. (Advanced) For $f(x, y) = x^2 + y^2 - 2x$, plot contour lines and show the iteration path of Gradient Descent.

MATLAB Commands and Examples

Gradient Descent Example

```
f = @(x) (x-2).^2;          % Function
df = @(x) 2*(x-2);          % Derivative

x = 10; alpha = 0.1; tol = 1e-6;
for k = 1:100
    x = x - alpha*df(x);
    if abs(df(x)) < tol, break; end
end
disp(['Minimum at x = ', num2str(x)])
```

Newton Method Example

```
f   = @(x) x.^4 - 3*x.^2 + 2;        % Function
df  = @(x) 4*x.^3 - 6*x;             % First derivative
d2f = @(x) 12*x.^2 - 6;              % Second derivative

x = 3; tol = 1e-6;
for k = 1:20
    x = x - df(x)/d2f(x);            % Newton update
    if abs(df(x)) < tol, break; end
end
disp(['Critical point at x = ', num2str(x)])
```