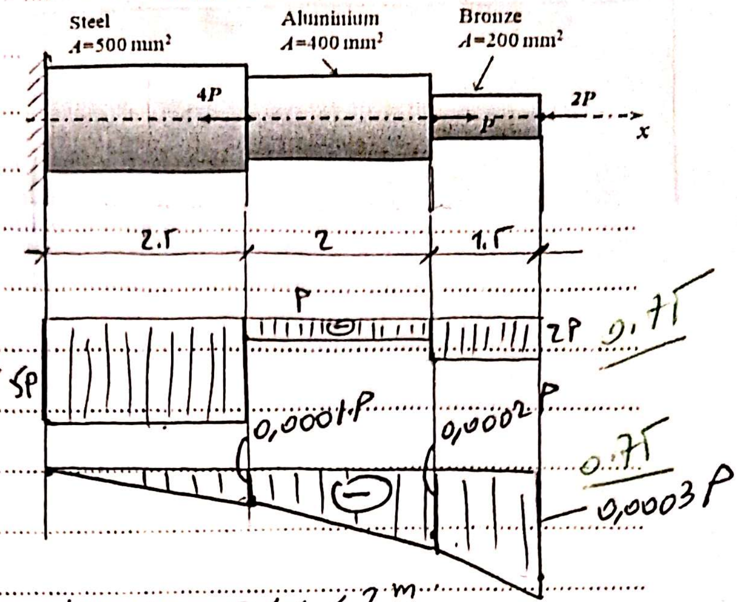
2nd year Bachlor's degree in Mechanical Engineering.

Final Exam in: Strength of Materials

Exercize 1: 6,5/6



1. Diagrams of N_x and u_x.

$R = ?$

$\sum F_x = 0 \Rightarrow -R - 4P + P - 2P = 0$

$\Rightarrow R = -5P$ 0,25

→ Normal force N_x along the bar.

• Section 1: $0 \leq x \leq 2.5$ m.

$\Rightarrow N_{x_1} = -5P < 0$ Compression. 0,5

• Section 2: $0 \leq x \leq 2$ m.

$N_{x_2} = -P < 0 \rightarrow$ Compression. 0,5

• Section 3: $0 \leq x \leq 1,5$ m.

$\rightarrow N_{x_3} = -2P < 0$ Compression. 0,5

→ Displacements u_x along the bar.

Section 1: $0 \leq x \leq 2,5$ m

$$\Delta L_1 = \int_0^x \frac{N_{x_1} dx}{E_1 A_1} = \frac{-5P}{E_1 A_1} x$$ 0,5

$E_1 = E_{st} = 200 \cdot 10^3$ MPa.

$A_1 = 500$ mm².

$\Delta L_1 = -0,005 \times P x$ [m]

$$\Delta L_1 = \begin{cases} 0 \rightarrow \text{for } x = 0 \\ 0,000125 P \rightarrow \text{for } x = 2,5 \end{cases}$$

• Section 2: $0 \leq x \leq 2$ m

$$\Delta L_2 = \Delta L_1 \Big|_{x=2,5} + \int_0^x \frac{N_{x_2} dx}{E_2 A_2}.$$ 0,5

$E_2 = E_{al} = 70$ mm², $A_2 = 400$ mm².

$$\Delta L_2 = 0,000125 P - \frac{P \cdot x}{E_2 A_2}$$

$$= \begin{cases} 0,000125 P & \text{for } x = 0 \\ 0,0001 \times P & \text{for } x = 2 \text{ m} \end{cases}$$

• Section 3: $0 \leq x \leq 1,5$ m

$$\Delta L_3 = \Delta L_2 \Big|_{x=2} + \int_0^x \frac{N_{x_3} dx}{E_3 A_3}$$ 0,5

$E_3 = E_{br} = 83 \times 10^3$ MPa ; $A_3 = 200$ mm²

$$\Delta L_3 = 0,0002 \times P - \frac{P \cdot x}{E_3 \cdot A_3}$$

$$= \begin{cases} 0,0002 \times P & \text{for } x = 0 \\ -0,0003 \times P & \text{for } x = 1,5 \text{ m} \end{cases}$$

2. $P_{adm} = ?$

Normal strength condition:

$$\sigma_{max} \leq [\sigma]$$

→ In steel Part.

$$\sigma_{max} = \frac{N_1}{A_1} \leq [\sigma]_{st} \Leftrightarrow \frac{5P}{A_1} \leq [\sigma$$

$$P \leq \frac{A_1}{5} [\sigma]_{st} \rightarrow P \leq \frac{500}{5} \times 1400$$

$\Rightarrow P \leq 14\,000$ N. (1) 0,5

→ In Aluminium Part:

$\sigma_{max} = \frac{N_2}{A_2} \leq [\sigma]_{Al}$ 0,5

$\Leftrightarrow \frac{P}{A_2} \leq [\sigma]_{Al} \to P \leq A_2 [\sigma]_{Al}$

$P \leq 400 \times 90 \to P \leq 36.000\, N$ (2)

→ In Bronze Part:

$\sigma_{max} = \frac{N_3}{A_3} \leq [\sigma]_{br}$ 0,5

$\Leftrightarrow \frac{2P}{A_3} \leq [\sigma]_{br} \to P \leq \frac{A_3}{2} [\sigma]_{br}$

$P \leq \frac{200}{2} \times 100 \to P \leq 10.000\, N$ (3)

From conditions 1, 2 and 3 we conclude that:

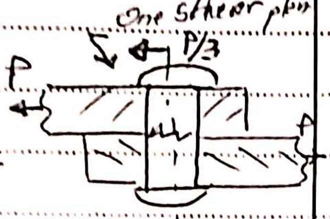
$P \leq 10.000\, N$.   0,25

So $P_{adm} = 10\ kN$.

Ex. 2: 6/6

Diameter d of the rivets?

• Shear strenght Condition



$\tau = \frac{T}{A} \leq [\tau]$

$T = P/3$; $A = \frac{\pi d^2}{4}$

0,5

$\Leftrightarrow \frac{4\,P/3}{\pi d^2} \leq [\tau] \Rightarrow d \geq \sqrt{\frac{4P}{3\pi [\tau]}}$

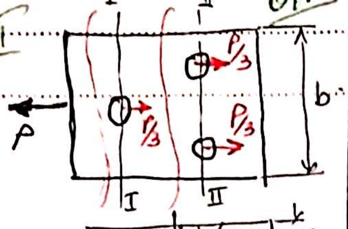
0,5

N.A.: $d \geq 5{,}95\ mm$ (1) 0,5

• Normal Strenght Condition (Tensil)

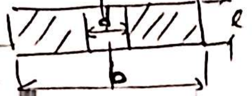
$\sigma_{max} \leq [\sigma]$; $\sigma = \frac{N}{A}$.

$\sigma_I = \frac{N_I}{A_I} = \frac{P}{e(b-d)}$ 0,5



$\frac{P}{e(b-d)} \leq [\sigma]$.

Section I-I



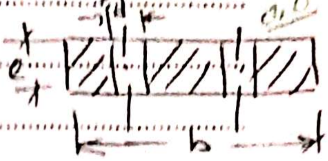
0,25

$\Rightarrow d \leq b - \frac{P}{e[\sigma]}$ 0,5

N.A. → $d \leq 17{,}5\ mm$ 0,5

$\sigma_{II} = \frac{N_{II}}{A_{II}} = \frac{\frac{2}{3}P}{e(b-2d)} \leq [\sigma]$

0,5



0,5

$\Leftrightarrow d \leq \frac{b}{2} - \frac{P}{3e[\sigma]}$ 0,5

N.A. → $d \leq 10{,}83$ (2) 0,5

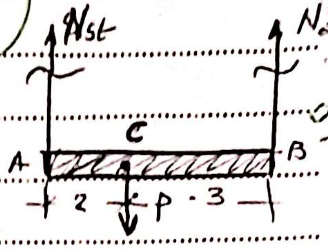
Conclusion: From conditions (1) and (2)

$\ldots \leq d \leq 10{,}83\ mm$ 0,5

We take: $d = 10{,}5\ mm$.

Exercize 3 A: 8/8

→ Internal force in the rods.



0,5

From static equilibrium equs:

• $\sum F_y = 0 \to N_{st} + N_{al} = P$ 0,5

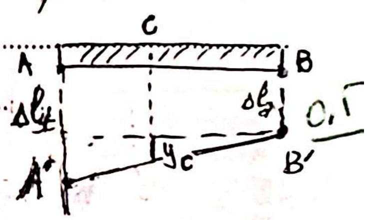
• $\sum M_A = 0 \to -P(2) + N_{al}(5) = 0$ 0,5

$N_{al} = \frac{2}{5}P = 20\,kN$; $N_{st} = \frac{3}{5}P = 30\,kN$

0,5   0,5

→ Vertical displacement of points A, B and C

$AA' = \Delta l_s = \frac{N_s \cdot L_s}{E_s \cdot A_s}$ 0,5

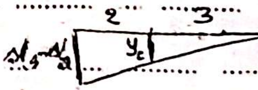


0,5

$BB' = \Delta l_a = \frac{N_a \cdot L_a}{E_a A_a}$

0,5

$CC' = \Delta l_a + y_c$ 0,5



0,5 $\frac{y_c}{3} = \frac{\Delta l_{st} - \Delta l_a}{5}$; $y_c = \frac{3}{5}(\Delta l_{st} - \Delta l_a)$

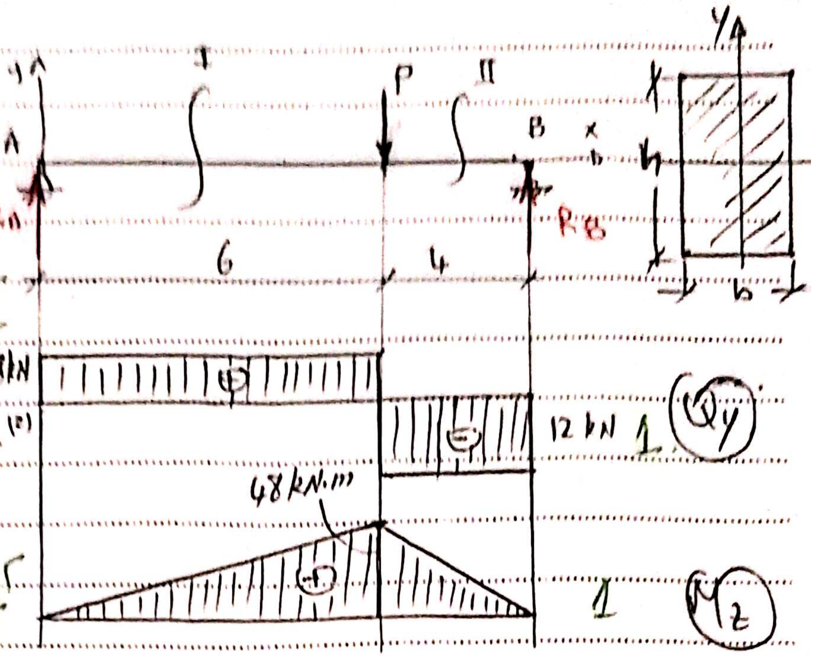
N.A.: $\Delta l_s = 2{,}5\ mm$ 0,5 ; $\Delta l_a = 2{,}3\ mm$ 0,5

$y_c = 0{,}08\ mm$ 0,5

$AA' = 2{,}5\ mm$; $BB' = 2{,}3\ mm$; $CC' = 2{,}38\ mm$

0,5   0,5   0,5

Exercice 3B: (8/8)



1. Reactions at the supports

Static Equilibrium eq^{ns}

- $\sum F_y = 0 \rightarrow R_A + R_B = P$ (1) 0.5
- $\sum M_A = 0 \rightarrow -P(6) + R_B(10) = 0$ (2) 0.5

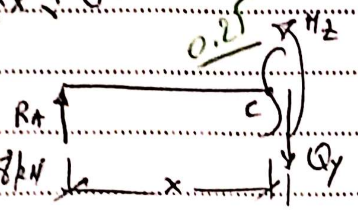
0.5 $\rightarrow R_B = \frac{3}{5} P$

(1) $\rightarrow R_A = \frac{2}{5} P$ 0.25

$R_B = 12$ kN ; $R_A = 8$ kN.

2. Q_y and M_z Eq^{ns}.

- Section I: $0 \leq x \leq 6$.

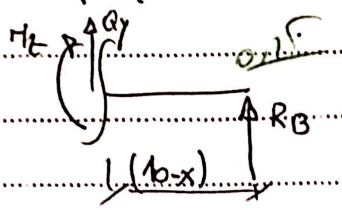


$Q_y = R_A = \frac{2}{5} P = 8$ kN 0.5

$-R_A \cdot x + M_z = 0 \rightarrow M_z = R_A \cdot x$.

0.5 $M_z = \begin{cases} 0 \rightarrow \text{for } x = 0. \\ 6R_A = 48 \text{ kN·m} \rightarrow \text{for } x = 6 \end{cases}$

- Section II: $6 \leq x \leq 10$



0.5 $Q_y = -R_B = -\frac{3}{5} P = -12$ kN.

0.5 $M_z = R_B \cdot (10-x) = \begin{cases} 0 \rightarrow \text{for } x = 10. \\ 4R_B \rightarrow \text{for } x = 6. \\ = 48 \text{ kN·m}. \end{cases}$ ✓

3. Bending strength condition

$\sigma_{x\,max} \leq [\sigma]$ 0.25

$\sigma_x = \frac{M_z}{I_z} y$ 0.5

$\sigma_{x\,max} = \frac{M_{z\,max}}{I_z} \cdot y_{max}$ 0.5

$y_{max} = \frac{h}{2} = \frac{16}{2} = 8$ mm.

from (M_z) $M_{z\,max} = 48$ kN·m $= 48 \cdot 10$

0.25 I_z: moment of inertia / z-z

0.5 $I_z = \frac{bh^3}{12} = 2730,66$ mm

N.A.:

$\sigma_{x\,max} \leq [\sigma]$. ✓

0.5