LBMU, SASF, Dep. ME **The 25/02/2025**

Problem Set: Numerical Solution of the Cauchy Problem (Initial Condition): Euler, Improved Euler (Heun), and 4th Order Runge-Kutta Methods

Exercise No. 1

The following Cauchy problem is given:

$$\begin{cases} y' = \dfrac{2y}{t} + t, \quad t \in [1, 1.6] \ \ and\ y \in [1,10] \\ \ y(1) = 1 \end{cases}$$

1. Verify the convergence of the numerical method for solving this problem.
2. Solve the equation using the Euler and Improved Euler (Heun) methods with a step size of h = 0.2.
3. Repeat the calculation using the Improved Euler (Heun) method with a step size of h = 0.1.
4. Plot and compare the results with the exact solution.

Exercise No. 2

The following Cauchy problem is given:

$$\begin{cases} y' = \cos(y) - t, \quad t, y \in [0,1] \\ y(0) = 0 \end{cases}$$

1. Verify the Lipschitz condition for this problem.
2. Use the RK4 method with a step size of h = 0.25 to solve this problem.
3. Plot the solution found.

Exercise No. 3 (Homework)

The following Cauchy problem is given:

$$\begin{cases} y' = y(t + y), \quad t \in [0, 1] \ \ et\ y \in [0,2] \\ y(0) = 1 \end{cases}$$

1. Verify the convergence of the numerical method for solving this problem.
2. Use the Euler and Modified Euler methods with a step size of h = 0.2 to solve this problem.
3. Repeat the calculation using the 4^{th} order Runge-Kutta method.
4. Plot the solutions on the same graph.

Let the following Cauchy problem be given:

$$\begin{cases} \ y' = -ty^2, \quad t \in [0, 1] \ \ et\ y \in [0.5,2] \\ y(0) = 1 \end{cases}$$

1. Verify the convergence of the numerical method for solving this problem.
2. Use the Runge-Kutta 4 method with a step size of h = 0.1 to solve this problem.
3. Compare with the exact solution at each calculation point.