1. Introduction

1.1 Definition of strength of materials

The strength of materials is a branch of mechanics also called mechanics of deformable solid bodies; it uses the notions of equilibrium of statics and the properties of materials, which are used for design and the sizing of structural parts or machine elements. It is the study of the stress and deformation of construction elements either in mechanical engineering or in civil engineering. It constitutes the essential tool for the engineer to create economical constructions which risk neither breaking nor excessively deforming under the actions applied to them.

1.2 Purpose of the study

Strength of materials covers the engineering methods used to calculate the strength, rigidity and stability of machine elements and constructions. We mean by :

Resistance : the capacity of a part to support and transmit the external loads imposed on it;

Rigidity: the part must not undergo excessive deformation when stressed;

Stability: the part must maintain its geometric integrity so that unstable conditions (buckling, spilling) are avoided**.**

A draft calculation based on strength of materials consists of:

- Choose the material constituting the element.
- Determine the functional dimensions of the element.
- Check the breaking resistance of the element.
- Optimize the cost of the part by changing shapes, dimensions, materials, etc.

1.3 General concepts

The design of different types of machines and various industrial constructions (bridges, transmission lines, hangars, ships, engines, turbines, power and nuclear power plants, machines, machine tools, etc.) cannot in any way depend move from basic knowledge of matrials resistance. However, despite the diversity of these structures, they can be reduced to relatively few major forms such as rods, beams, plates (Figure 2) and massive bodies.

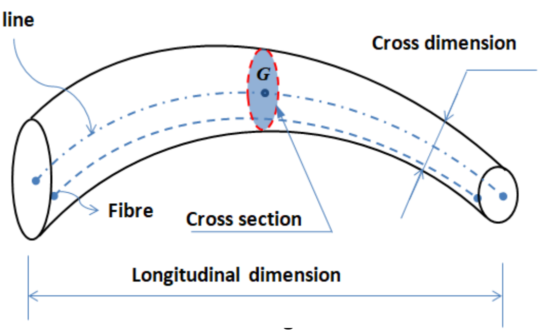


Figure 1

- ***The Main assumptions***

In its current use, resistance of materials uses the following assumptions:

- ***Continuity of material***: metal with fibrous structure, or sometimes granular; the distances between these fibers or these grains are very small compared to the dimensions of the smallest parts;
- ***elastic:*** the material returns to its initial shape after a loading/unloading cycle;
- ***linear:*** *deformations are proportional to stresses;*
- ***homogeneous:*** the material is of the same nature throughout its mass;
- ***isotropic:*** the properties of the material are identical in all directions**.**

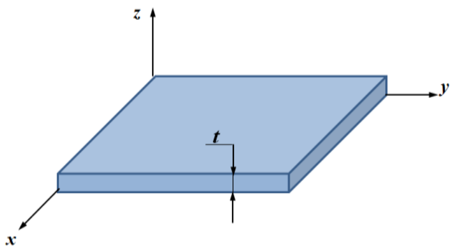


Figure 2

The issue is

- in small deformations*;*

- static (no dynamic effects);

- isothermal (no temperature change)

On the other hand, throughout the content of the course, the following hypotheses will be retained:
Navier – Bernoulli assumption: the sections perpendicular to the axis of the bar remain plane and perpendicular during the deformation process (see Fig. 1).

Saint – Venant assumption: the results of the resistance only apply validly at a distance sufficiently far from the region of application of the concentrated forces. Indeed, we cannot, with the study equations, calculate the local deformations around a point of application of a force.

1.4 Types of loads

1.4.1 Concepts of external forces and internal forces

External forces or loads are the forces of interaction between an element considered and the bodies which are in contact with it; they are exerted by objects external to the system. Internal forces are those which are exerted by objects internal to the system, ***they act under the action of external forces***.
The study of the internal effects of forces acting on a body constitutes one of the concerns of RDM. We will see later, and depending on the type of request, the analysis and determination of these mechanical actions.

1.4.2 Types of mechanical loads applied to the systems considered

In addition to the external forces applied to the system, the reactions at the supports are concentrated forces
A load distributed over the surface that can be brought back to the principal plane, that is to say a load distributed over a line, is called a load per unit length, most often measured in N/m (q in Figure 3).

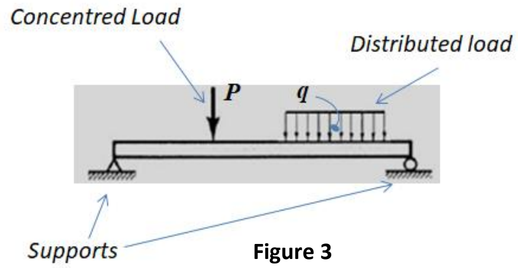


Figure 3

In the case of a uniformly distributed load, the q diagram is rectangular. If it is uniformly variable, the q diagram is triangular.

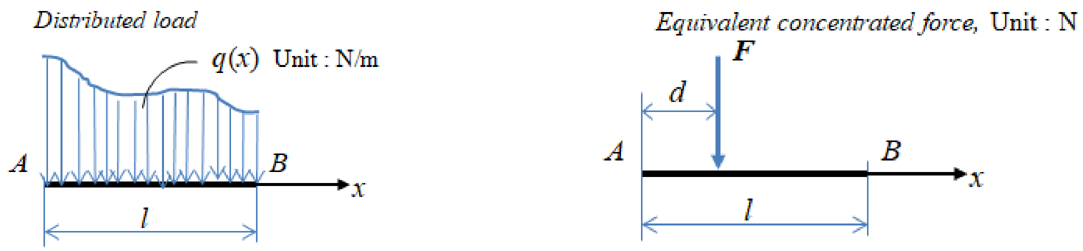
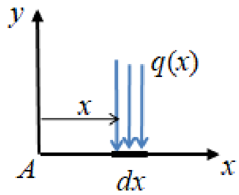


Figure 4

Changing to a force equivalent to a single-direction distributed load is a commonly accepted method.
The charge having a fixed direction we can write:



$$F = \int q(x)dx \qquad (1)$$

The resultant of a distributed load is numerically equal to the area of its diagram and is applied to the center of gravity of the latter.

3.1.1 Static Equilibrium (Rappel)

We recall that all vector quantities are often represented in relation to a direct orthonormal reference frame with respective axes Ox, Oy and Oz.

The first equilibrium condition for the static equilibrium of a rigid body expresses the equilibrium in translation, we write:

$$\sum \boldsymbol{F} = \boldsymbol{0} \quad (2)$$

This vector equation is equivalent to the three scalar equations for the force components:

$$\sum F_{ix} = 0\,;\ \sum F_{iy} = 0\,;\ \sum F_{iz} = 0\,; \quad (3)$$

It should be noted that in (2) and (3) it is indeed the sum of all external forces. The second condition for the static equilibrium of a rigid body expresses rotational equilibrium:

$$\sum \boldsymbol{M}(\boldsymbol{F}) = \boldsymbol{0} \quad (4)$$

Again, equation (4) is equivalent to three scalar equations for the vector components of the moment:

$$\sum M_{ix} = 0\,;\ \sum M_{iy} = 0\,;\ \sum M_{iz} = 0\,; \quad (5)$$

4. Axially load Bar

Consider at the start an initially straight metal bar of constant cross section, loaded at its ends by a pair of oppositely directed collinear forces coinciding with the longitudinal axis of the bar and acting through the centroid of each cross section.

For static equilibrium the magnitudes of the forces must be equal.
If the forces are directed away from the bar, the bar is said to be in tension; if they are directed toward the bar, a state of compression exists. These two conditions are illustrated in Fig. 5.

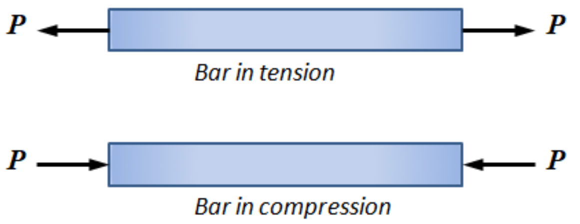


Figure 5 : Axially loaded bar

Cutting plan *a-a* is normal of the axe of the bar. The originally internal forces now become external with respect to the remaining portion of the body (Fig. 6a).
For equilibrium of the portion to the left this ''effect'' must be a horizontal force of magnitude P. However, this force $\boldsymbol{P}$ **acting normal to the cross section *a-a*** is actually the resultant of distributed forces acting over this cross section in a direction normal to it.

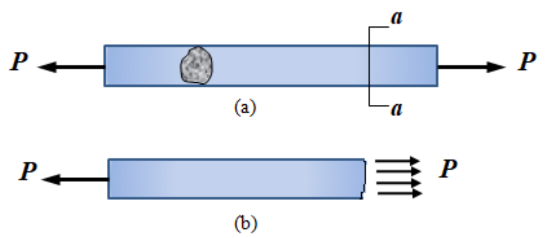


Figure 6 : Internal forces

4.1 Normal stress

Consider the bar in tension of the figure 5. Knowing the original cross-sectional area, the normal stress, denoted by $\boldsymbol{\sigma}$, may be obtained for any value of the axial load by the use of the relation :

$$\sigma = \frac{P}{A} \quad (6)$$

The intensity of normal force per unit area is termed the ***normal stress*** and is expressed in units of force per unit area, N/m^2.
If the forces applied to the ends of the bar are such that the bar is in tension, then *tensile stresses* are set up in the bar; if the bar is in compression we have *compressive stresses*. The line of action of the applied end forces passes through the centroid of each cross section of the bar.

Units : MPa or N/mm^2 (Mega-Pascal, 1 MPa = 1 N/mm^2).

4.2 Normal strain

Let us suppose that the bar of Fig. 5 has tensile forces gradually applied to the ends. The elongation per unit length, which is termed normal strain and denoted by ε, may be found by dividing the total elongation ΔL by the length L, i.e.,

$$\varepsilon = \frac{\Delta L}{L} \tag{7}$$

The strain is usually expressed in units of meters per meter and consequently is *dimensionless*.

4.3 Normal stress - normal strain relation

4.3.1 Stress – strain curve

As the axial load in Fig. 6 (bar in tension) is gradually increased, the total elongation over the bar length is measured at each increment of load and this is continued until fracture of the ***specimen*** takes place. This is the ***tensile test***.

Having obtained numerous pairs of values of normal stress σ and normal strain ε, experimental data may be plotted with these quantities considered as ordinate and abscissa, respectively. This is the ***stress-strain curve*** (Figure 7) or diagram of the material for this type of loading.

It is worth noting that stress-strain diagrams assume widely differing forms for various materials. The tensile test alone allows defining the common mechanical characteristics used in mechanics of materials.

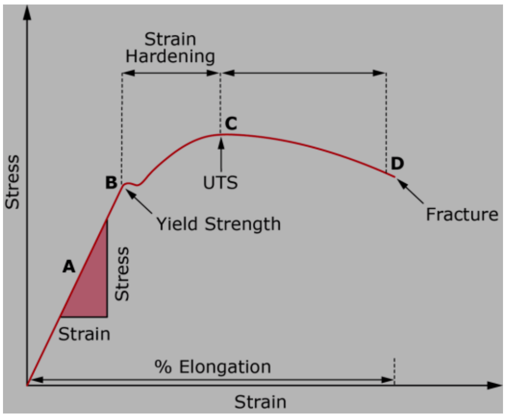


Figure 7 : Stress – Strain curve.

An interesting detail is given in the video of the following link : https://youtu.be/FyhIezLB0wQ.

4.3.2 Hooke's Law

As it shown in Figure 8, the relation between stress and strain is linear. This linear relation between elongation and the axial force causing it is called ***Hooke's law***. To describe this initial linear range of action of the material we may consequently write

$$\sigma = E\varepsilon \tag{8}$$

where E denotes the slope of the straight-line portion AB of the curve in Fig.8.

The quantity E, i.e., the ratio of the unit stress to the unit strain, is the *modulus of elasticity of the material* in tension, or, as it is often called, *Young's modulus*. Values of E for various engineering materials are tabulated in handbooks.

It is noted that the behavior of materials under load as discussed in this course is restricted to the linear region of the stress-strain curve.

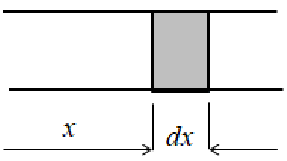
Using equations 7 and 8, we demonstrate that the total elongation of a bar of length L and constant section A stressed in tension by the force P is given by:

$$\Delta L = \frac{PL}{EA} \tag{9}$$

In the general case where the internal normal force changes along the length of the bar, and also the case of a variable section, considering an element of length dx we can write:

$$\sigma(x) = \frac{N(x)}{A(x)}$$

$$\varepsilon_x = \frac{\Delta x}{dx} = \frac{\sigma_x}{E} = \frac{N(x)}{EA(x)}$$



The elementary elongation is:

$$\Delta l = \int_x \frac{N(x)}{EA(x)} dx \tag{10}$$

The integration is done according to the length of each part, and then the summation according to the parts of the bar.

$$\Delta L = \sum \int_x \frac{N(x)}{EA(x)} dx \tag{11}$$

4.3.3 Thermomechanical behavior

Temperature changes in a structure give rise to internal stresses, just as do applied loads. The thermal strain due to a temperature change $\boldsymbol{\Delta T}$ is

$$\varepsilon_T = \alpha \cdot \Delta T \tag{12}$$

The coefficient of expansion (usually denoted by α) is defined as the change of length per unit length of a straight bar subject to a temperature change of one degree. The value of this coefficient is independent of the unit of length but does depend upon the temperature scale used.

Example : for steel material $\alpha = 1.2\ 10^{-5} / °C$.

Equation 10 represents the elongation of the bar loaded in tension by P. Under a loading in force and temperature, the ***Hooke-Duhamel law*** is

$$\Delta l = \frac{PL}{EA} + \alpha \Delta T \tag{13}$$

The stresses generated by a temperature difference are called thermal stresses.

4.4 Ultimate Strength Condition

The condition of tensile strength is simply written as

$$\sigma_{\max} \leq [\sigma] \tag{14}$$

Where $[\sigma]$ or σ_{alow} is the allowable stress of the material.

4.5 Poisson's Ratio

When a bar is subjected to a simple tensile loading there is an increase in length of the bar in the direction of the load, but a decrease in the lateral dimensions perpendicular to the load. The ratio of the strain in the lateral direction to that in the axial direction is defined as Poisson's ratio. For most metals it lies in the range 0.25 to 0.35.

$$\nu = \frac{\text{Lateral strain}}{\text{Axial strain}} \tag{15}$$

5. SI System of Units and conversion to English Units

The SI system of units is actually used worldwide and it's also called as international system of units. The table below shows the units of the most well-known physical quantities and their conversion into EU.

Quantity	Symbol	SI Units	English Units	To Convert from English to SI Units Multiply by
Length	L	m	ft	0.3048
Mass	m	kg	lbm	0.4536
Time	t	s	sec	1
Area	A	m^2	ft^2	0.09290
Volume	V	m^3	ft^3	0.02832
Velocity	V	m/s	ft/sec	0.3048
Acceleration	a	m/s^2	ft/sec^2	0.3048
Angular velocity	ω	rad/s	rad/sec	1
		rad/s	rpm	9.55
Force, Weight	F, W	N	lbf	4.448
Density	ρ	kg/m^3	lbm/ft^3	16.02
Specific weight	γ	N/m^3	lbf/ft^3	157.1
Pressure, stress	p, σ, τ	kPa	psi	6.895
Work, Energy	W, E, U	J	ft-lbf	1.356
Power	W	W	ft-lbf/sec	1.356
		W	hp	746