LBMU, SASF, Dep. ME **The 16/02/2025**

Problem Set: Numerical Integration using Trapezoidal, Simpson's, and Quadrature Formulas

<u>Exercise No. 1:</u>

1. Calculate the approximate value of the integral $I = \int_0^1 e^{-x^2}\,dx = \frac{\sqrt{\pi}}{2}\mathrm{erf}(1) \approx 0.7468$ using:
   a. The Trapezoidal method with 1, 4, and 8 intervals.
   b. Simpson's method with 2, 6, and 8 intervals.
2. Calculate the error committed by each method and discuss.

<u>Exercise No. 2:</u>

Consider the numerical calculation of the integral $I = \int_0^3 \frac{xdx}{x^2+1} = \frac{\ln(10)}{2} \approx 1.1513.$

1. Find the step size h such that the error does not exceed 10^{-4} for both the Trapezoidal and Simpson's methods.
2. Calculate the integral using these two methods with the found step size.
3. Calculate the exact value of the integral and make the comparison.

<u>Exercise No. 3:</u>

We want to calculate $\int_1^3 (xe^x - 1)dx = 2(e^3 - 1) \approx 38.1710$ using a quadrature formula of the form: $\int_1^3 f(x)dx = A_0 f(1) + A_1 f(2) + A_2 f(3)$.

1. Write the system of equations that gives the coefficients A_i.
2. Solve the system and calculate the integral.
3. Evaluate the error in this calculation.
4. Calculate the exact value of the integral and make the comparison.

<u>Exercise No. 4 (Homework):</u>

a) Use Simpson's method to calculate the integral $I = \int_1^2 \frac{dx}{x} = \ln(2) \approx 0.6932$ with 2, 4, 6, and 8 intervals.

1. Calculate the error committed in this calculation.
2. Find the integral using the form $\int_1^2 \frac{dx}{x} dx = A_0 f(1) + A_1 f\left(\frac{4}{3}\right) + A_1 f\left(\frac{5}{3}\right) + A_2 f(2)$.
3. Calculate the exact value of the integral and make the comparison.

b) Consider the numerical calculation of the integral $I = \int_{-3}^3 \frac{dx}{x^2+1} dx = 2tan^{-1}(3) = 2.4981$ using Simpson's method.

1. Find the step size h such that the error does not exceed 0.0001.
2. Calculate the integral using h=1.
3. Calculate the exact value of the integral and make the comparison.