Problem Set No. 2: Interpolation with Newton and Lagrange Polynomials

Exercise 1:

An experiment yields the distance as a function of time, as shown in the following table:

t(s)	0	2	3	5
x(m)	-1	2	9	87

We want to find an approximation of x(t) on the interval [0,5].

1. Calculate the coefficients of the Lagrange polynomials based on the table.
2. Find the Lagrange polynomial that approximates x(t).
3. Calculate the divided difference table for the given data.
4. Find the Newton polynomials that approximate x(t) to degrees 1, 2, and 3.
5. Plot the found polynomials on the same graph, including the Lagrange polynomial.

Exercise 2:

Let the function be $f(x) = 2\sin\left(\frac{\pi x}{6}\right)$, with x in radians.

1. Find the Lagrange polynomial based on the following nodes (points): 0, 3, and 6.
2. Increase the number of points: 0, 1.5, 3, 4.5, and 6. Recalculate the Lagrange polynomial.
3. Find the error term e(x) in each case.
4. Evaluate the error when replacing *f(2)* with P(2) in both cases.
5. Plot the function f and the found polynomials on the same graph.
6.

Exercise 3:

We have to calculate the integral of $f(x) = I = \int_0^1 f(x)\,dx = \int_0^1 e^{-x^2}\,dx$.

1. Find the Newton polynomial for the function f based on the points $0, \frac{1}{2}, \text{and } 1$.
2. Calculate the integral I using the found polynomial.
3. Evaluate the error committed in this calculation.

Homework Exercise 1:

A) Redo Exercise 2 using the Newton polynomial and Exercise 3 using the Lagrange method.

B) Let the function be $\boldsymbol{f(x) = |x| - \cos(x)}$, with x in radians.

1. Calculate the divided difference table for the function f at the points $\boldsymbol{-1, -\frac{1}{2}, 0, \frac{1}{2}, \text{and } 1}$.
2. Calculate the Newton polynomials for the function f.
3. Find the interpolation error term.

C) Let the function f be defined by $\boldsymbol{f(x) = x^2 - \cos(x)}$, with x in radians. We want to approximate f(x) by an interpolation polynomial.

1. Calculate the Newton polynomial of the function f based on the points -2, -1, 0, 1, and 2.
2. Find the expression for the interpolation error term.