TD 03

Exercise 1

Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$

1. Calculate A^2 and A^3. Evaluate $A^3 - A^2 + A - I$.
2. Express A^{-1} in terms of A^2, A, and I.
3. Express A^4 in terms of A^2, A, and I.

Exercise 2

Let $A = \begin{bmatrix} 3 & 0 & 1 \\ -1 & 3 & -2 \\ -1 & 1 & 0 \end{bmatrix}$

Calculate $(A - 2I)^3$, then deduce that A is invertible and find A^{-1} in terms of I, A, and A^2.

Exercise 3

Consider the matrix A defined as :

$$A = \begin{bmatrix} 5 & 6 & -3 \\ -18 & -19 & 9 \\ -30 & -30 & 14 \end{bmatrix}$$

1. Is A invertible ? If yes, determine its inverse A^{-1}.
2. Calculate $A^2 - A - 2I_3 = 0$, where I_3 is the identity matrix.

Exercise 4

Consider the matrix A defined as :

$$A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

1. Find $a, b \in \mathbb{R}$ such that $A^2 = aI_3 + bA$.
2. Deduce that A is invertible and determine its inverse.

Exercise 5

Consider the matrix associated with the linear transformation f defined on $\mathbb{R}^3$ with respect to the canonical basis :

$$A = \begin{bmatrix} 1 & -1 & 5 \\ 3 & 0 & 2 \\ 1 & 1 & 4 \end{bmatrix}$$

1. Determine the linear application f.
2. Find $ker\ f$ and $Im\ f$, along with their dimensions. Is f bijective ?
3. Let $S = \{v_1 = (1, 1, 1), v_2 = (1, 0, 1), v_3 = (2, -1, 0)\}$:
   (a) Show that S is a basis for $\mathbb{R}^3$.
   (b) Find the matrix associated with f with respect to the basis S.