TD 02

Exercise 1

Let the application f defied by $\mathbb{R}^2$ in $\mathbb{R}^2$ by :

$$f\ (x, y) = (x + y,\ x - y).$$

1. Prove that f is linear.
2. Find $ker\ f$, and $Im\ f$ and find the dimensions, f Is it bijective ?
3. Determine $f \circ f$.

Exercise 2

Let f be the function defined from $\mathbb{R}^2$ to $\mathbb{R}^2$ by :

$$f(x, y) = (2x - 4y,\quad x - 2y).$$

1. Show that f is linear.
2. Determine $ker\ f$ and $Im\ f$, and give their dimensions. Is f bijective ?

Exercise 3

Let $f : \mathbb{R}^3 \to \mathbb{R}^2$ be defined by

$$f(x, y, z) = (x + y + z, -x + 2y + 2z).$$

Let $\beta = (e_1, e_2, e_3)$ be the canonical basis of $\mathbb{R}^3$ and $\beta' = (f_1, f_2)$ be the canonical basis of $\mathbb{R}^2$.

1. Show that f is a linear function.
2. Provide a basis and the dimension of $ker\ f$ and a basis and the dimension of $Im\ f$.

Exercise 4

Let f be the linear transformation $f : \mathbb{R}^3 \to \mathbb{R}^3$ defined by :

$$f\ \ (x_1, x_2, x_3) = (x_1 - x_3,\ 2x_1 + x_2 - 3x_3,\ \ -x_2 + 2x_3).$$

Let (e_1, e_2, e_3) be the canonical basis of $\mathbb{R}^3$.

1. Compute $f\ \ (e_1)$, $f\ \ (e_2)$, and $f\ \ (e_3)$.
2. Determine the coordinates of $f\ \ (e_1)$, $f\ \ (e_2)$, and $f\ \ (e_3)$ in the canonical basis.
3. Compute a basis for $ker\ f$ and a basis for $Im\ f$.

Exercise 5

Let f be the endomorphism of $\mathbb{R}^3$ whose image of the canonical basis $\beta = (e_1, e_2, e_3)$ is :

$$\begin{cases} f(e_1) = -7e_1 - 6e_2, \\ f(e_2) = 8e_1 + 7e_2, \\ f(e_3) = 6e_1 + 6e_2 - e_3. \end{cases}$$

1. For any vector $x = x_1e_1 + x_2e_2 + x_3e_3$, determine $f \circ f(x)$.
2. Deduce that f is invertible (i.e., bijective) and determine f^{-1}.