ULBM, SASF, ME Dep. **2025**

Problem Set No. 1: Non-linear Equation Solving Methods

Exercise 1: (Locating roots of an equation of the form f(x) = 0) Find graphically the intervals that include the roots of the following equations:
$\boldsymbol{x^2 - 10x + 23 = 0}$, $\boldsymbol{e^x - x - 2 = 0}$, $\cos(x) - x + 1 = 0$ and $ln(x) - 5 + x = 0$.

Exercise 2: (Bisection Method)

1. Locate the roots of the equations: $x^3 + 4x^2 - 10 = 0$ and $\underline{\boldsymbol{xsin(x) - 1 = 0.}}$
2. Use the bisection method to calculate the root that belongs to the interval [1, 2] for the first equation and [0, 2] for the second, with a precision of 0.001.

Exercise 3: (Successive Approximations Method)

1. For equations 1 and 2 from Exercise 1, write all possible forms of x = g(x).
2. Using the intervals found in Exercise 1, check the convergence of the successive approximations' method for these forms.
3. Calculate the roots of equations 1 and 2 using the successive approximations method with a precision of 0.005.

Exercise 4: (Newton-Raphson Method)

a) Given the equation $\boldsymbol{xe^x - 3 = 0}$ with $x \in [0, 2]$.

1. Verify the convergence conditions for the Newton-Raphson method.
2. Calculate the solution of the equation with a precision $\varepsilon = 10^{-5}$.

b) Establish a Newton-Raphson formula that allows the calculation of de $\frac{1}{a^m}$ and $\sqrt[m]{\boldsymbol{a}}$ with a and m > 1 without using the root and division. Test with the calculation of $\frac{1}{4}$ and $\sqrt{4}$.

Exercise 5: (Homework - Written quizzes will be taken from homework exercises)

A) Consider the following equation: $\boldsymbol{|x|e^x - 1 = 0 \quad \text{for } x \neq 0}$.

1. Use the graphical method to find the number of roots of this equation. Verify the intervals found by calculation.
2. Use the bisection method to find the roots of this equation with a precision of 0.01.
3. Deduce the negative root of $\boldsymbol{|x|e^{|x|} - 1 = 0 \quad \text{for } x \neq 0}$..
4. We want to calculate this negative root $\overline{\boldsymbol{x}} \in [\boldsymbol{-1, -0.25}]$ using the Newton-Raphson method.
   a. Verify the convergence conditions of the method.
   b. Calculate the root given $\boldsymbol{x_0 = -0.500}$ and $\boldsymbol{\varepsilon = 0.001}$..

B)

1. Locate the roots of equations 3 and 4 from Exercise 1 and equation 1 from Exercise 2.
2. Use the bisection method to find the roots of these equations with a precision of 0.001.
3. Find the roots using the successive approximations method.
4. Find these roots using the Newton-Raphson method.