Inverse Trigonometric Functions: $\arctan(x)$, $\arcsin(x)$, and $\arccos(x)$

1. Arctangent ($\arctan(x)$)

Definition

The arctangent function is the inverse of the tangent function. It is defined as:

$$y = \arctan(x) \quad \text{if and only if} \quad \tan(y) = x \quad \text{and} \quad -\frac{\pi}{2} < y < \frac{\pi}{2}.$$

Here, $y = \arctan(x)$ means that y is the angle whose tangent is x and y lies within the interval $-\frac{\pi}{2} < y < \frac{\pi}{2}$.

Properties

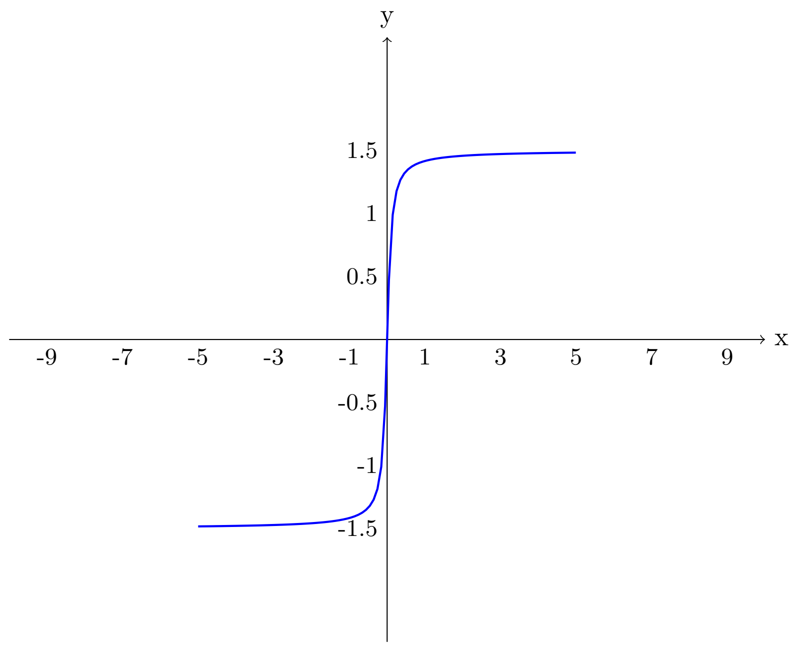
- **Domain**: $\mathbb{R}$ (all real numbers).
- **Range**: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.
- **Monotonicity**: The arctangent function is strictly increasing.
- **Asymptotes**:

$$\lim_{x\to\infty} \arctan(x) = \frac{\pi}{2}, \quad \lim_{x\to-\infty} \arctan(x) = -\frac{\pi}{2}.$$

Graph

The graph of $y = \arctan(x)$ has the following characteristics:

- It passes through the origin: $\arctan(0) = 0$.
- The graph approaches $\frac{\pi}{2}$ as $x \to \infty$ and $-\frac{\pi}{2}$ as $x \to -\infty$.



Derivative

The derivative of $\arctan(x)$ is given by:

$$\frac{d}{dx}\arctan(x) = \frac{1}{1+x^2}.$$

2. Arcsine ($\arcsin(x)$)

Definition

The arcsine function is the inverse of the sine function. It is defined as:

$$y = \arcsin(x) \quad \text{if and only if} \quad \sin(y) = x \quad \text{and} \quad -\frac{\pi}{2} \leq y \leq \frac{\pi}{2}.$$

Properties

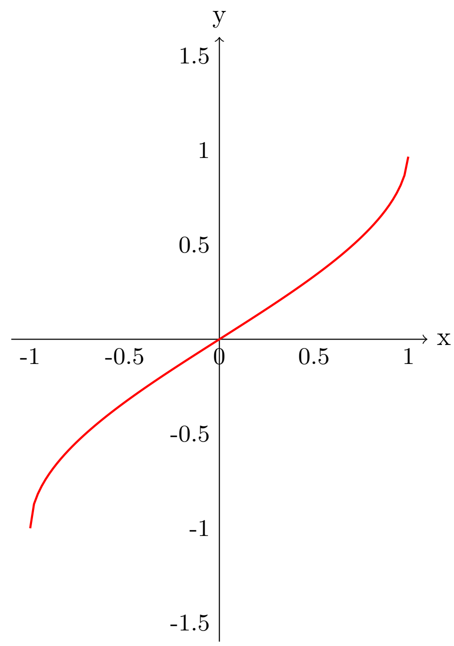
- **Domain**: $[-1, 1]$.
- **Range**: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.
- **Monotonicity**: The arcsine function is strictly increasing.
- **Asymptotes**:

$$\lim_{x\to 1}\arcsin(x) = \frac{\pi}{2}, \quad \lim_{x\to -1}\arcsin(x) = -\frac{\pi}{2}.$$

Graph

The graph of $y = \arcsin(x)$ has the following characteristics:

- It passes through the origin: $\arcsin(0) = 0$.
- The graph approaches $\frac{\pi}{2}$ as $x \to 1$ and $-\frac{\pi}{2}$ as $x \to -1$.



Derivative

The derivative of $\arcsin(x)$ is given by:

$$\frac{d}{dx}\arcsin(x) = \frac{1}{\sqrt{1-x^2}}.$$

3. Arccosine ($\arccos(x)$)

Definition

The arccosine function is the inverse of the cosine function. It is defined as:

$$y = \arccos(x) \quad \text{if and only if} \quad \cos(y) = x \quad \text{and} \quad 0 \leq y \leq \pi.$$

Properties

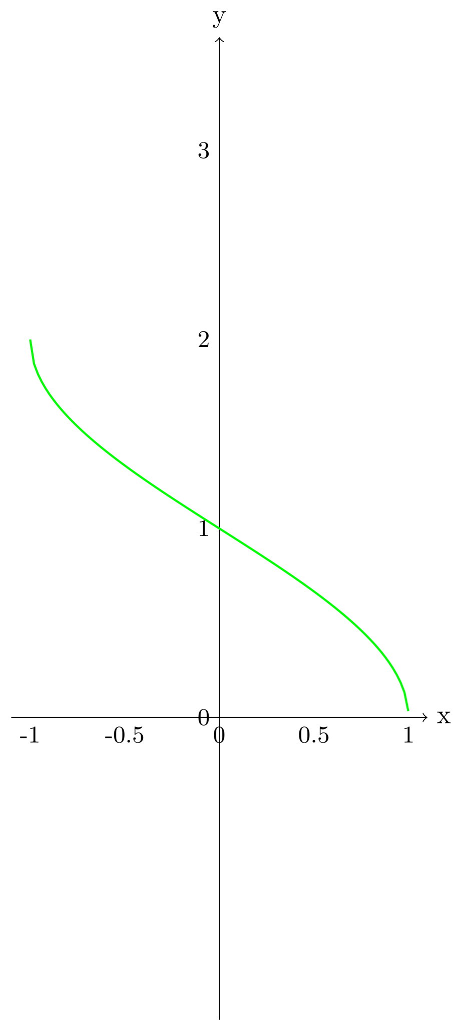
- **Domain**: $[-1, 1]$.
- **Range**: $[0, \pi]$.
- **Monotonicity**: The arccosine function is strictly decreasing.
- **Asymptotes**:

$$\lim_{x \to 1} \arccos(x) = 0, \quad \lim_{x \to -1} \arccos(x) = \pi.$$

Graph

The graph of $y = \arccos(x)$ has the following characteristics:

- It passes through $(1, 0)$ and $(-1, \pi)$.



Derivative

The derivative of $\arccos(x)$ is given by:

$$\frac{d}{dx}\arccos(x) = -\frac{1}{\sqrt{1-x^2}}.$$