Exponential and Logarithmic Functions

Exponential Functions

An exponential function is of the form:

$$f(x) = a^x$$

where:

- $a > 0$ and $a \neq 1$ (base),
- x is the exponent.

Properties of Exponential Functions

1. **Domain:** $x \in \mathbb{R}$.
2. **Range:** $f(x) > 0$.
3. **Intercept:** The graph passes through $(0, 1)$, since $a^0 = 1$.
4. **Growth and Decay:**
   - If $a > 1$, the function is increasing (growth).
   - If $0 < a < 1$, the function is decreasing (decay).
5. **Laws of Exponents:**

$$\begin{aligned} a^m \cdot a^n &= a^{m+n}, \\ \frac{a^m}{a^n} &= a^{m-n}, \\ (a^m)^n &= a^{m \cdot n}, \\ a^0 &= 1, \\ a^{-n} &= \frac{1}{a^n}. \end{aligned}$$

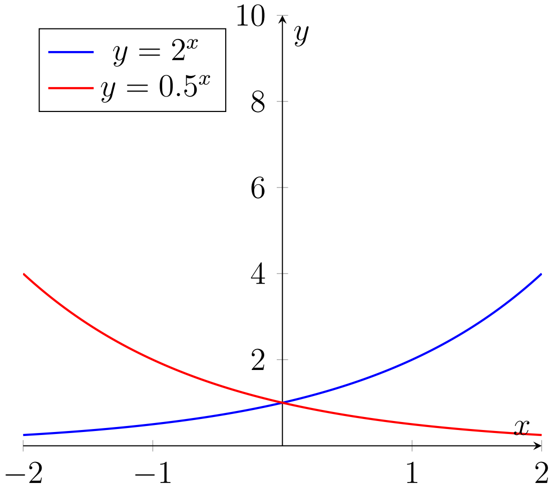
6. **Derivative:**

$$\frac{d}{dx}a^x = a^x \ln(a).$$

7. **Integral:**

$$\int a^x \, dx = \frac{a^x}{\ln(a)} + C.$$

Graph of Exponential Functions



Logarithmic Functions

A logarithmic function is the inverse of the exponential function. It is defined as:

$$y = \log_a(x) \quad \Leftrightarrow \quad a^y = x,$$

where $a > 0$, $a \neq 1$, and $x > 0$.

Properties of Logarithmic Functions

1. **Domain:** $x > 0$.

2. **Range:** $y \in \mathbb{R}$.

3. **Intercept:** The graph passes through $(1, 0)$, since $\log_a(1) = 0$.

4. **Laws of Logarithms:**

$$\log_a(xy) = \log_a(x) + \log_a(y),$$
$$\log_a\left(\frac{x}{y}\right) = \log_a(x) - \log_a(y),$$
$$\log_a(x^n) = n \cdot \log_a(x),$$
$$\log_a(a) = 1,$$
$$\log_a(1) = 0.$$

5. **Change of Base Formula:**

$$\log_a(x) = \frac{\log_b(x)}{\log_b(a)}.$$

6. **Derivative:**

$$\frac{d}{dx}\log_a(x) = \frac{1}{x\ln(a)}.$$

7. **Integral:**

$$\int \log_a(x)\,dx = x\ln(x) - x\ln(a) + C.$$

Graph of Logarithmic Functions