Solution of short test n°02 of Maths 03 (A)

Answer:

1) $\sum_{n\geq 1} \frac{2n+5}{n!}\, x^n$, $a_n = \frac{2n+5}{n!}$, $\lim_{n\to+\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to+\infty} \frac{2n+7}{(n+1)!}\,\frac{n!}{2n+5}$

$$= \lim_{n\to+\infty} \frac{2n+7}{2n+5}\,\frac{1}{n+1} = 0$$

Then : $\Re = \frac{1}{l} = +\infty$ and $I = \mathbb{R}$.

2) $S = \sum_{n\geq 1} \frac{2n+5}{n!}\, x^n = \sum_{n\geq 1} \frac{2n}{n!}\, x^n + \sum_{n\geq 1} \frac{5}{n!}\, x^n = 2\sum_{n\geq 1} \frac{n}{n!}\, x^n + 5\sum_{n\geq 1} \frac{x^n}{n!}$

we have : $\sum_{n\geq 0} \frac{x^n}{n!} = e^x$ By derivation : $\sum_{n\geq 1} \frac{nx^{n-1}}{n!} = e^x$ Times x :

We obtain : $\sum_{n\geq 1} \frac{nx^n}{n!} = xe^x$

$S = 2xe^x + 5(e^x - 1) = (2x+5)e^x - 5.$

3) $f(x) = \frac{2x+2}{3x^2-2x-1} = \frac{2x+2}{(3x+1)(x-1)} = \frac{a}{3x+1} + \frac{b}{x-1} = \frac{-1}{3x+1} + \frac{1}{x-1}$

$$f(x) = \frac{-1}{1+3x} - \frac{1}{1-x} = -\sum_{n\geq 0}(-3x)^n - \sum_{n\geq 0} x^n = \sum_{n\geq 0}\left((-1)^{n+1}3^n - 1\right)x^n$$