Solution of short test n°02 of Maths 03 (B)

Answer:

1) $\sum_{n\geq 1}\frac{(-1)^n}{n\,2^{n+1}}\,x^n \quad , \quad a_n=\frac{(-1)^n}{n\,2^{n+1}} \quad , \quad \lim_{n\to+\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to+\infty}\frac{1}{2}\frac{n}{n+1}=\frac{1}{2}.$

$$\Re = \frac{1}{l} = 2$$

$x = 2$, $\quad \sum_{n\geq 1}\frac{(-1)^n}{n\,2^{n+1}}\,2^n = \sum_{n\geq 1}\frac{(-1)^n}{2n}$ Alternating serie ,

Leinitz : $\begin{cases} v_n = \frac{1}{2n} \text{ Decreasing} \\ \lim_{n\to+\infty} v_n = 0 \end{cases}$ Then $\sum_{n\geq 1}\frac{(-1)^n}{2n}$ converges.

$x = -2$, $\quad \sum_{n\geq 1}\frac{(-1)^n}{n\,2^{n+1}}\,(-2)^n = \sum_{n\geq 1}\frac{1}{2n}$ Harmonic serie diverges.

Then : the interval of convergence is : $I =]-2 \quad 2]$.

2) $S = \sum_{n\geq 1}\frac{(-1)^n}{n\,2^{n+1}}\,x^n = \frac{1}{2}\sum_{n\geq 1}\frac{1}{n}\left(-\frac{x}{2}\right)^n$

We have : $\sum_{n\geq 1}\left(-\frac{x}{2}\right)^n = \frac{1}{1+\frac{x}{2}} = \frac{2}{2+x} \;\Rightarrow \frac{1}{2}\sum_{n\geq 1}\frac{(-1)^n}{2^n}\,x^n = \frac{1}{2+x}$

Divide by x : $\sum_{n\geq 1}\frac{(-1)^n}{2^{n+1}}\,x^{n-1} = \frac{1}{x(2+x)}$

By integration : $\sum_{n\geq 1}\frac{(-1)^n}{n2^{n+1}}\,x^n = \int \frac{dx}{x(2+x)} = \frac{1}{2}\int\left(\frac{1}{x}-\frac{1}{2+x}\right)dx = \frac{1}{2}ln\left(\frac{x}{2+x}\right).$

3) $f(x) = \frac{3}{2x^2-x-1} = \frac{3}{(2x+1)(x-1)} = \frac{-2}{2x+1}+\frac{1}{x-1} = \frac{-2}{1+2x}-\frac{1}{1-x}$

$$f(x) = -2\sum_{n\geq 0}(-2x)^n - \sum_{n\geq 0}x^n = \sum_{n\geq 0}((-1)^{n+1}2^{n+1}-1)x^n$$