Solution of question n°2 12/12/2024

1) differential equation of motion "7 pts"

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot\theta}\right) - \frac{\partial L}{\partial \theta} = -\frac{\partial D}{\partial \dot\theta} \quad (0{,}25)\ (0{,}25)\ (0{,}25)$$

$$\frac{\partial L}{\partial \dot\theta} = \frac{1}{2} M R^2 \dot\theta \ (0{,}25) \ ; \quad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot\theta}\right) = \frac{1}{2} M R^2 \ddot\theta \ (0{,}25) \ , \quad D = \frac{1}{2}\alpha R^2 \dot\theta^2 \ (0{,}25)$$

$$\frac{\partial L}{\partial \theta} = -K R^2 \theta \ (0{,}25) \ , \quad \frac{\partial D}{\partial \dot\theta} = \alpha R^2 \dot\theta \ (0{,}25)$$

$$\frac{1}{2} M R^2 \ddot\theta + \alpha R^2 \dot\theta + K R^2 \theta = 0 \quad (0{,}25)\ (0{,}25)\ (0{,}25)$$

$$(0{,}25)\ \ddot\theta + \frac{2\alpha}{M}\dot\theta + \frac{2K}{M}\theta = 0 \quad (0{,}25)$$

$$\ddot\theta + 2\delta\dot\theta + \omega_0^2\theta = 0 \Rightarrow \delta = \frac{\alpha}{M} \ (0{,}25) \ ; \quad \omega_0 = \sqrt{\frac{2K}{M}} \ (0{,}25)$$

2) The solution (0,25)

$$\theta(t) = A e^{-\delta t}\cos(\omega_a t + \varphi) \ (0{,}25) \ ; \quad \omega_a = \sqrt{\omega_0^2 - \delta^2} \ (0{,}25)$$

Initial conditions: $\theta(0) = A e^{-\delta \cdot 0}\cos(\omega_a \cdot 0 + \varphi) = 0$ (0,25)

$$\Rightarrow \cos(\varphi) = 0 \Rightarrow \varphi = \pm\frac{\pi}{2} \ (0{,}25)$$

$$\dot\theta(t) = -\delta A e^{-\delta t}\cos\left(\omega_a t - \frac{\pi}{2}\right) \ (0{,}25) - \omega_a A e^{-\delta t}\sin\left(\omega_a t - \frac{\pi}{2}\right) \ (0{,}25)$$

$$\dot\theta(0) = -\delta A e^{-\delta \cdot 0}\underbrace{\cancel{\cos\left(\omega_a \cdot 0 - \frac{\pi}{2}\right)}}_{0} \ (0{,}25) - \omega_a A e^{-\delta 0}\underbrace{\cancel{\sin\left(\omega_a \cdot 0 - \frac{\pi}{2}\right)}}_{-1} \ (0{,}25) = \dot\theta_0$$

$$\Rightarrow \omega_a \cdot A = \dot\theta_0 \Rightarrow A = \frac{\dot\theta_0}{\omega_a} \ (0{,}25) \Rightarrow \theta(t) = \frac{\dot\theta_0}{\omega_a} e^{-\delta t}\cos\left(\omega_a t - \frac{\pi}{2}\right) \ (0{,}25)$$

3) Forced system

$$\ddot\theta + \frac{2\alpha}{M}\dot\theta + \frac{2K}{M}\theta = (0{,}5)\ \left(2F_0 R / M R^2\right)\cos\Omega t$$

or if $\varphi = \frac{\pi}{2} \Rightarrow \theta(t) = \frac{-\dot\theta_0}{\ } e^{-\delta t}\cos(\omega t + \pi)$