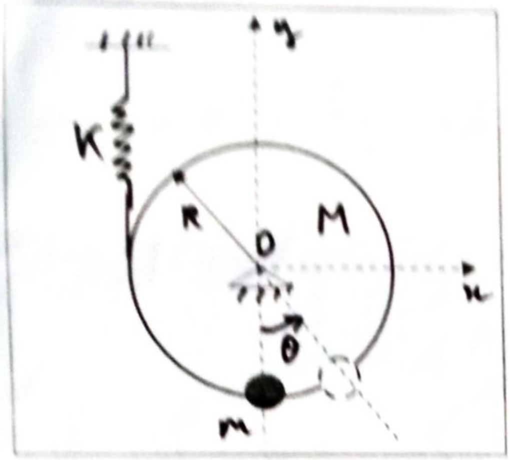
Version 2

First and last name :	Group	Registration:

$\frac{6}{6}$



Consider the mechanical system composed of a disk (M, R) which can rotate around its center O. A point mass **m** is fixed at a distance **R** from the center **O** of the disk. A spring with a stiffness constant **k** connects the left generatrix of the disc to the frame.

- Calculate the kinetic energy T, the potential energy U and the Lagrangian L =T-U of the system
- Determine the differential equation of motion.
- Deduce the system's own pulsation.

Answer :

$T_M = \frac{1}{4} M R^2 \dot{\theta}^2$ 0,5

$m \begin{cases} x_m = R\sin\theta & 0{,}25 \\ y_m = -R\cos\theta & 0{,}25 \end{cases}$ $\begin{cases} \dot{x}_m = R\dot{\theta}\cos\theta & 0{,}25 \\ \dot{y}_m = R\dot{\theta}\sin\theta & 0{,}25 \end{cases}$ $\Rightarrow T_m = \frac{1}{2} m R^2 \dot{\theta}^2$ 0,25

$T_t = T_M + T_m = \left(\frac{1}{4}M + \frac{1}{2}m\right) R^2 \dot{\theta}^2$ 0,25

$\frac{-\partial V_m}{\partial y} = -mg$ 0,25 ; $V_m = mg\, y_m = -mgR\cos\theta$ 0,25

$V_k = \frac{1}{2} K y_k^2 = \frac{1}{2} K R^2 \sin^2\theta$ 0,25

$V_t = V_m + V_k = -mgR\cos\theta + \frac{1}{2} K R^2 \sin^2\theta$ 0,25

$L = T_t - V_t = \left(\frac{1}{4}M + \frac{1}{2}m\right) R^2\dot{\theta}^2 + mgR\cos\theta - \frac{1}{2} K R^2 \sin^2\theta$ 0,25 0,25

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot\theta}\right) - \frac{\partial L}{\partial \theta} = 0$$

$$\frac{\partial L}{\partial \dot\theta} = \left(\frac{1}{2}M + m\right)R^2\dot\theta \quad ; \quad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot\theta}\right) = \left(\frac{1}{2}M + m\right)R^2\ddot\theta$$

0,25 0,25

$$\frac{\partial L}{\partial \theta} = -mgR\sin\theta - KR^2\sin\theta\cos\theta \quad ; \sin\theta \simeq \theta, \cos\theta \simeq$$

0,25 0,25

$$\frac{\partial L}{\partial \theta} = -(mgR + KR^2)\theta$$

0,25

The equation in the form $\ddot\theta + \omega_0^2\theta = 0$.

$$\left(\frac{1}{2}M + m\right)R^2\ddot\theta + (mgR + KR^2)\theta = 0.$$

0,25 0,25

$$\ddot\theta + \left(\frac{mgR + KR^2}{\frac{1}{2}M + m}\right)\theta = 0$$

0,25 0,25

$$\omega_0 = \sqrt{\frac{mgR + KR^2}{\frac{1}{2}M + m}}$$

0,25

Version 1

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$\frac{6}{6}$

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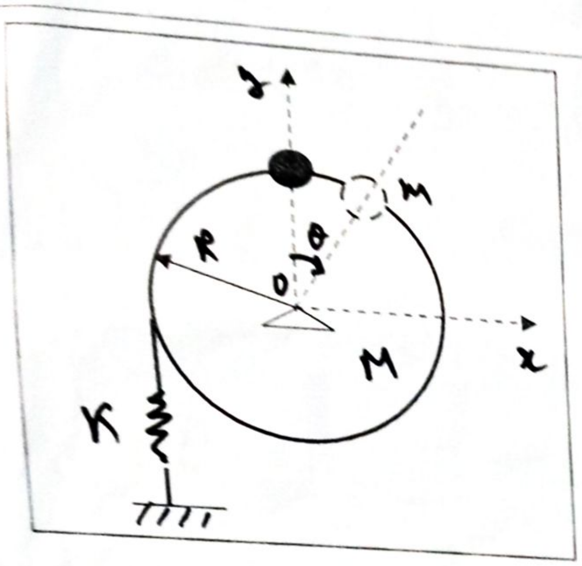

Consider the mechanical system composed of a disk **(M, R)** *which can rotate around its center* **O**. *A point mass* **m** *is fixed at a distance* **R** *from the center* **O** *of the disk. A spring with a stiffness constant* **k** *connects the left generatrix of the disc to the frame*

- *Calculate the kinetic energy* **T**, *the potential energy* **U** *and the Lagrangian* **L =T-U** *of the system*
- *Determine the differential equation of motion.*
- *Deduce the system's own pulsation.*

Answer :

0,5

$$T_M = \frac{1}{4} M R^2 \dot{\theta}^2$$

$$m \begin{cases} x_m = R \sin\theta \ (0,25) \\ y_m = R\cos\theta \ (0,25) \end{cases} \Rightarrow \begin{cases} \dot{x}_m = R\dot{\theta}\cos\theta \ (0,25) \\ \dot{y}_m = -R\dot{\theta}\sin\theta \ (0,25) \end{cases} \Rightarrow T_m = \frac{1}{2} m R^2 \dot{\theta}^2 \ (0,25)$$

0,25

$$T_t = T_M + T_m = \left(\frac{1}{4}M + \frac{1}{2}m\right) R^2 \dot{\theta}^2$$

$$\frac{-\partial V_m}{\partial y} = -mg \ (0,25) \quad ; \quad V_m = mg\, y_m = mgR\cos\theta \ (0,25)$$

0,25

$$V_k = \frac{1}{2} K y_k^2 = \frac{1}{2} K R^2 \sin^2\theta$$

0,25

$$V_t = V_m + V_k = mgR\cos\theta + \frac{1}{2} K R^2 \sin^2\theta$$

0,5 0,25

$$L = T_t - V_t = \left(\frac{1}{4}M + \frac{1}{2}m\right) R^2 \dot{\theta}^2 - mgR\cos\theta - \frac{1}{2} K R^2 \sin^2\theta$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot\theta}\right) - \frac{\partial L}{\partial \theta} = 0$$

$$\frac{\partial L}{\partial \dot\theta} = \left(\frac{1}{2}M + m\right)R^2\dot\theta \quad ; \quad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot\theta}\right) = \left(\frac{1}{2}M+m\right)R^2\ddot\theta$$

$$\frac{\partial L}{\partial \theta} = mgR\sin\theta - KR^2\sin\theta\cos\theta \quad , \sin\theta \simeq \theta , \cos\theta \simeq 1$$

$$= (mgR - KR^2)\theta$$

the equation in the form:

$$\ddot\theta + \omega_0^2\,\theta = 0$$

$$\left(\frac{1}{2}M+m\right)R^2\ddot\theta + (KR^2 - mgR)\theta = 0$$

$$\ddot\theta + \frac{KR^2 - mgR}{\frac{1}{2}M+m}\,\theta = 0$$

$$\omega_0 = \sqrt{\frac{KR^2 - mgR}{\frac{1}{2}M+m}}$$