Electromechanical Analogies

Introduction

Mechanical systems can be represented using equivalent electrical circuits. A mechanical system and an electrical system are said to be analogous if the differential equations governing their behavior are identical. When this equivalence is achieved, the corresponding terms in the differential equations are called analogous.

There are two types of analogies between mechanical and electrical systems:

- The **force-voltage analogy**, also known as the **mass-inductance analogy**, which is the most commonly used.
- The **force-current analogy**, also referred to as the **mass-capacitance analogy**.

Mechanical-Electrical Correspondence Table

The complete correspondence between mechanical and electrical quantities can be summarized as follows:

Mechanical Quantities	Electrical Quantities	Relationship
Position (x)	Electric charge (Q)	$x \leftrightarrow Q$
Velocity ($\dot{x} = \frac{dx}{dt}$)	Electric current ($i = \frac{dQ}{dt}$)	$\dot{x} \leftrightarrow i$
Force (F)	Voltage (v)	$F \leftrightarrow v$
Spring stiffness (k)	Inverse of capacitance ($1/C$)	$k \leftrightarrow 1/C$
Damping coefficient (c)	Electrical resistance (R)	$c \leftrightarrow R$

Table 1: Mechanical-Electrical Correspondence Table

This table summarizes all key correspondences between mechanical and electrical systems based on their analogous behaviors.

Single Degree of Freedom Systems

Solved Exercise

The mass-damper-spring system presented above is governed by the following differential equation:

$$m\frac{d^2x}{dt^2} + \alpha\frac{dx}{dt} + kx = F$$

This equation can also be expressed in terms of velocity $\dot{x}$ as:

$$m\frac{d\dot{x}}{dt} + \alpha\dot{x} + k\int \dot{x}\,dt = F$$

Furthermore, applying Kirchhoff's second law yields the differential equation governing the RLC series circuit illustrated above:

$$L\frac{di}{dt} + Ri + \frac{1}{C}\int i\,dt = e$$

These two differential equations are of the same nature, meaning the corresponding physical systems are referred to as *analogous*.

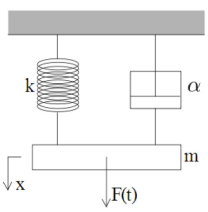

Figure 1: 1D.O.F Mechanical system

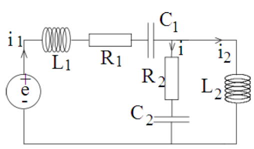

Figure 2: Equivalent electrical circuit

Two Degrees of Freedom System

Solved Exercise

Using the force-voltage analogy, establish the electrical system analogous to the two-degree-of-freedom mechanical system shown below.

Solution

A methodical approach is necessary to derive the analogous electrical circuit. The steps are as follows:

1. **Establish the system of differential equations:**

$$m_1\ddot{x}_1 + (\alpha_1 + \alpha_2)\dot{x}_1 + (k_1 + k_2)x_1 - \alpha_2\dot{x}_2 - k_2x_2 = F$$

$$m_2\ddot{x}_2 + \alpha_2\dot{x}_2 + k_2x_2 - \alpha_2\dot{x}_1 - k_2x_1 = 0$$

2. **Group terms by factoring each quantity associated with a mechanical element:**

$$m_1\ddot{x}_1 + \alpha_1\dot{x}_1 + k_1x_1 + \alpha_2(\dot{x}_1 - \dot{x}_2) + k_2(x_1 - x_2) = F$$

$$m_2\ddot{x}_2 - \alpha_2(\dot{x}_1 - \dot{x}_2) - k_2(x_1 - x_2) = 0$$

3. **Express the system in integro-differential form:**

$$m_1\frac{d\dot{x}_1}{dt} + \alpha_1\dot{x}_1 + k_1\int \dot{x}_1 dt + \alpha_2(\dot{x}_1 - \dot{x}_2) + k_2\int(\dot{x}_1 - \dot{x}_2)dt = F$$

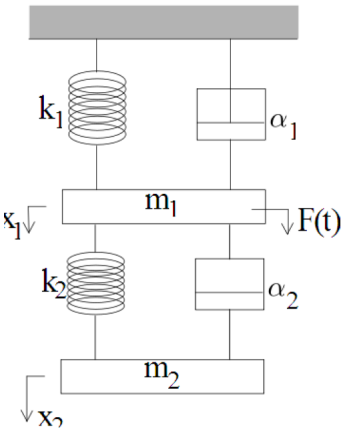

Figure 3: 2D.O.F Mechanical system

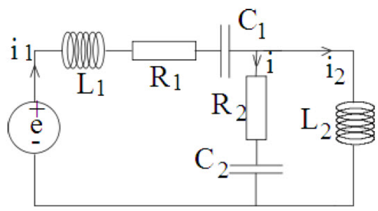

Figure 4: Equivalent electrical circuit