

## Exercice : 2

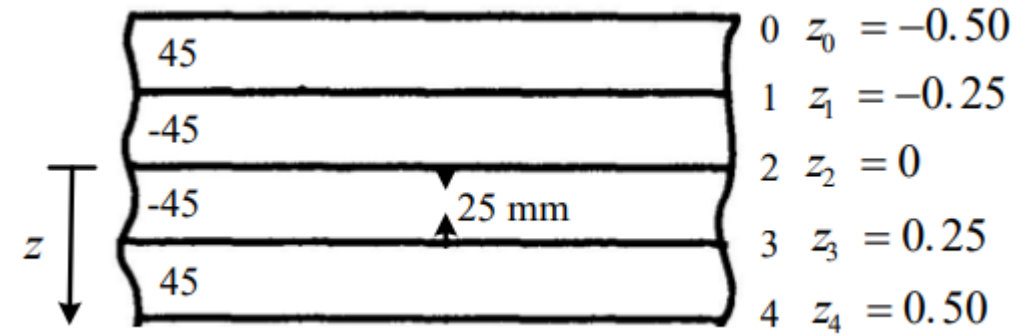
$$[45/-45/-45/45] \Rightarrow Np = 4 \text{ plis}$$

Unidirectionnel : AS/3501 d'après le tableau

$$\begin{cases} E_1 = 138 \text{ GPa} \\ E_2 = 9 \text{ GPa} \end{cases} \quad \begin{cases} G_{12} = 6.9 \text{ GPa} \\ \nu_{12} = 0.3 \end{cases}$$

$$\nu_{21} = \nu_{12} \frac{E_2}{E_1} = 0.3 \times \frac{9}{138} = 0.0196$$

### Paramétrage



Constantes élastiques fondamentales des composites usuels [Gibson]

Composites	$E_1$	$E_2$	$G_{12}$	$\nu_{12}$	$\nu_f$
	Msi (GPa)	Msi (GPa)	Msi (GPa)		
T300/934 Graphite/epoxy	19.0 (131)	1.5 (10.3)	1.0 (6.9)	0.22	0.65
AS/3501 Graphite/epoxy	20.0 (138)	1.3 (9.0)	1.0 (6.9)	0.3	0.65
p-100/ERL 1962 pitch graphite/epoxy	68.0 (468.9)	0.9 (6.2)	0.81 (5.58)	0.31	0.62
Kevlar® 49/934 aramid/epoxy	11.0 (75.8)	0.8 (5.5)	0.33 (2.3)	0.34	0.65
Scotchply® 1002 E-glass/epoxy	5.6 (38.6)	1.2 (8.27)	0.6 (4.14)	0.26	0.45
Boron/5505 Boron/epoxy	29.6 (204)	2.68 (18.5)	0.81 (5.59)	0.23	0.5
Spectra® 900/826 Polyethylene/epoxy	4.45 (30.7)	0.51 (3.52)	0.21 (1.45)	0.32	0.65
E-glass/470-36 E-glass/vinylester	3.54 (24.4)	1.0 (6.87)	0.42 (2.89)	0.32	0.30

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➤ Matrice de rigidité dans le plan 1-2

$$[Q] = \begin{bmatrix} \frac{E_1}{1-\nu_{12}\nu_{21}} & \frac{\nu_{12}E_2}{1-\nu_{12}\nu_{21}} & 0 \\ & \frac{E_2}{1-\nu_{12}\nu_{21}} & 0 \\ \text{SYM} & & G_{12} \end{bmatrix} = \begin{bmatrix} 138.81 & 2.72 & 0 \\ 2.72 & 9.05 & 0 \\ 0 & 0 & 6.9 \end{bmatrix}$$

➤ Matrices de rigidité des plis par rapport au plan moyen du stratifié  
Plan  $x-y$

$$[\bar{Q}] = [T(\theta)]^{-1} [Q] [T^1(\theta)]$$

$$[Q]_1 = [Q]_4 = \begin{bmatrix} 45.22 & 31.42 & 32.44 \\ 31.42 & 45.22 & 32.44 \\ 32.44 & 32.44 & 35.6 \end{bmatrix} \text{ GPa}$$

$$[Q]_2 = [Q]_3 = \begin{bmatrix} 45.22 & 31.42 & -32.44 \\ 31.42 & 45.22 & -32.44 \\ -32.44 & -32.44 & 35.6 \end{bmatrix} \text{ GPa}$$

➤ Matrice  $ABD$

$$A_{ij} = \sum_{k=1}^4 (\bar{Q}_{ij})_k (z_k - z_{k-1})$$

$$B_{ij} = \frac{1}{2} \sum_{k=1}^4 (\bar{Q}_{ij})_k (z_k^2 - z_{k-1}^2)$$

$$D_{ij} = \frac{1}{3} \sum_{k=1}^4 (\bar{Q}_{ij})_k (z_k^3 - z_{k-1}^3)$$

$$[A] = \begin{bmatrix} 45.22 & 31.42 & 0 \\ 31.42 & 45.22 & 0 \\ 0 & 0 & 35.6 \end{bmatrix} \text{GPa} - \text{mm}$$

$$[B] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{GPa} - \text{mm}^2$$

$$[D] = \begin{bmatrix} 3.77 & 2.62 & 2.03 \\ 2.62 & 3.77 & 2.03 \\ 2.03 & 2.03 & 2.97 \end{bmatrix} \text{GPa} - \text{mm}^3$$

$$ABD = \begin{bmatrix} 45.22 & 31.42 & 0 & 0 & 0 & 0 \\ 31.42 & 45.22 & 0 & 0 & 0 & 0 \\ 0 & 0 & 35.6 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3.77 & 2.62 & 2.03 \\ 0 & 0 & 0 & 2.62 & 3.77 & 2.03 \\ 0 & 0 & 0 & 2.03 & 2.03 & 2.97 \end{bmatrix}$$

## Exercice : 4

Stratifié [45 / -45]  $\Rightarrow Np = 2$  plis

Soit  $t = 0.25 \text{ mm}$ , l'épaisseur des plis.

$$A_{ij} = \sum_{k=1}^2 (\bar{Q}_{ij})_k (z_k - z_{k-1}) = (\bar{Q}_{ij})_1 \cdot t + (\bar{Q}_{ij})_2 \cdot t$$

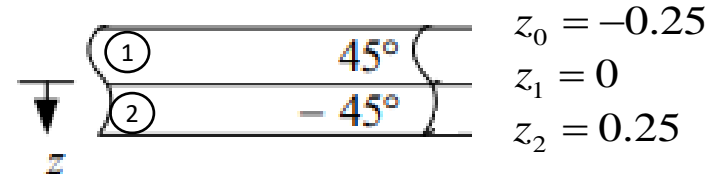
$$[A] = \begin{bmatrix} 22.61 & 15.71 & 0 \\ 15.71 & 22.61 & 0 \\ 0 & 0 & 17.8 \end{bmatrix} \text{ GPa} \cdot \text{mm} \text{ - ou } \frac{\text{N}}{\text{mm}^2} \times \text{mm} = \frac{\text{N}}{\text{mm}}$$

$$B_{ij} = \frac{1}{2} \sum_{k=1}^2 (\bar{Q}_{ij})_k (z_k^2 - z_{k-1}^2) = -\frac{1}{2} (\bar{Q}_{ij})_1 \cdot t^2 + \frac{1}{2} (\bar{Q}_{ij})_2 \cdot t^2$$

$$[B] = \begin{bmatrix} 0 & 0 & -2.027 \\ 0 & 0 & -2.027 \\ -2.027 & -2.027 & 0 \end{bmatrix} \text{ GPa} \cdot \text{mm}^2 \text{ ou } 10^3 \text{ N}$$

$$D_{ij} = \frac{1}{3} \sum_{k=1}^2 (\bar{Q}_{ij})_k (z_k^3 - z_{k-1}^3) = \frac{1}{3} (\bar{Q}_{ij})_1 \cdot t^3 + \frac{1}{3} (\bar{Q}_{ij})_2 \cdot t^3$$

$$[D] = \begin{bmatrix} 0.471 & 0.327 & 0 \\ 0.327 & 0.471 & 0 \\ 0 & 0 & 0.371 \end{bmatrix} \text{ GPa} \cdot \text{mm}^3$$



$$[Q]_1 = \begin{bmatrix} 45.22 & 31.42 & 32.44 \\ 31.42 & 45.22 & 32.44 \\ 32.44 & 32.44 & 35.6 \end{bmatrix} \text{ GPa}$$

$$[Q]_2 = \begin{bmatrix} 45.22 & 31.42 & -32.44 \\ 31.42 & 45.22 & -32.44 \\ -32.44 & -32.44 & 35.6 \end{bmatrix} \text{ GPa}$$

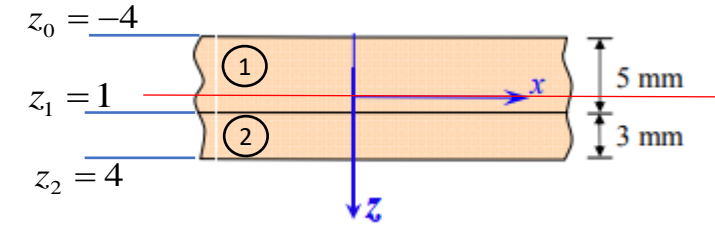
$$ABD = \begin{bmatrix} 22.61 & 15.71 & 0 & 0 & 0 & -2.027 \\ 15.71 & 22.61 & 0 & 0 & 0 & -2.027 \\ 0 & 0 & 17.8 & -2.027 & -2.027 & 0 \\ 0 & 0 & -2.027 & 0.471 & 0.327 & 0 \\ 0 & 0 & -2.027 & 0.327 & 0.471 & 0 \\ -2.027 & -2.027 & 0 & 0 & 0 & 0.371 \end{bmatrix}$$

## Exercice : 6

Stratifié [0 / 45]  $\Rightarrow Np = 2$  plis

Epaisseurs des plis :  $t_1 = 5$  mm et  $t_2 = 3$  mm.

Caractéristiques du matériau des plis :  $E_1 = 46$  GPa ;  $E_2 = 10$  GPa ;  $G_{12} = 4.6$  GPa ;  $\nu_{12} = 0.31$ .



$$[Q] = \begin{bmatrix} \frac{E_1}{1-\nu_{12}\nu_{21}} & \frac{\nu_{12}E_2}{1-\nu_{12}\nu_{21}} & 0 \\ & \frac{E_2}{1-\nu_{12}\nu_{21}} & 0 \\ \text{SYM} & & G_{12} \end{bmatrix} = \begin{bmatrix} 46.982 & 3.166 & 0 \\ 3.166 & 10.213 & 0 \\ 0 & 0 & 4.6 \end{bmatrix} \text{ GPa}$$

$$[\bar{Q}]_1 = [Q]$$

$$[\bar{Q}]_2 = \begin{bmatrix} 20.482 & 11.282 & 9.192 \\ 11.282 & 20.482 & 9.192 \\ 9.192 & 9.192 & 12.716 \end{bmatrix} \text{ GPa}$$

Calcul de la matrice  $ABD$

$$A_{ij} = \sum_{k=1}^4 (\bar{Q}_{ij})_k (z_k - z_{k-1})$$

$$B_{ij} = \frac{1}{2} \sum_{k=1}^4 (\bar{Q}_{ij})_k (z_k^2 - z_{k-1}^2)$$

$$D_{ij} = \frac{1}{3} \sum_{k=1}^4 (\bar{Q}_{ij})_k (z_k^3 - z_{k-1}^3)$$

➤ **Remarque** : Il s'agit d'un stratifié quelconque (aucune symétrie), la matrice  $ABD$  est pleine.