Larbi Ben Mh'idi University Oum el Boughi
Academic Year: 2023/2024
Department of Computer Science
Second Year L.M.D-Computer Science
Module: Mathematical Logic
Module Coordinator: Dr. Boussaha K.

TD Series No = 02 part 02 (propositional logic : semantics)

Conjunctive and disjunctive normal forms with conversion algorithms

Exercise 01:

Given the following logical expressions, determine if they are in CNF, DNF

1. (A∨B)∧(C∨D)
2. A∧(B∨C)∧(D)
3. (A∧B)∨(C∧D)
4. (A∨B)∧(C)
5. A∨(B∧C)

Exercice 02:

Using the conversion algorithm,find the disjunctive normal forms

a. (A ∨ B ∨ C) ∧ (C ∨ ¬A)

b. (A ∨ B) ∧ (C ∨ D)

c . ¬((A ∨ B) → C)

d. A∧(B∨C)

e. (A∨B)∧C

f. ¬A∨(B∧C)

Exercice 03:

Using the conversion algorithm, find the conjunctive normal forms

a. (A ∨ B) → (C ∧ D)
b. (A ∨ (¬B ∧ (C ∨ (¬D ∧ E))))
c. A ↔ (B ∧ ¬C)
d. A∨(B∧C)
e. (A∧B)∨C
f. A→(B∨C)
g. ¬(A ∨B)

Exercice 04:

1. demonstrate that the following formula is valid :

F = $(a \land \neg b) \lor (\neg a \land \neg (b \lor c)) \lor (\neg c \land b) \lor (b \land c \land a) \lor (c \land \neg a)$.

2. Deduce (without proof) what can be said about the validity of the following formula A:
A = $(a \rightarrow b) \land (\neg a \rightarrow (b \lor c)) \land (\neg c \rightarrow \neg b) \land ((b \land c) \rightarrow \neg a) \land (c \rightarrow a)$.

Solution

Exercise 01:

CNF, DNF

1. $(A \lor B) \land (C \lor D)$ CNF
2. $A \land (B \lor C) \land (D)$ CNF
3. $(A \land B) \lor (C \land D)$ DNF
4. $(A \lor B) \land (C)$ CNF
5. $A \lor (B \land C)$ DNF

Exercise n=02

a) $(A \lor B \lor C) \land (C \lor \neg A)$

$\equiv (A \land (C \lor \neg A)) \lor (B \land (C \lor \neg A)) \lor (C \land (C \lor \neg A)$ distributivity.

$\equiv ((A \land C) \lor (A \land \neg A)) \lor ((B \land C) \lor (B \land \neg A)) \lor ((C \land C) \lor (C \land \neg A))$ distributivity.

$\equiv (A \land C) \lor ((B \land C) \lor (B \land \neg A)) \lor (C \lor (C \land \neg A))$ we delete $(A \land \neg A)$ = False

$\equiv (A \land C) \lor ((B \land C) \lor (B \land \neg A)) \lor C$ we a remplace $(C \lor (C \land \neg A))$ with C

Absorption law .
$\equiv (A \land C) \lor (B \land C) \lor (B \land \neg A) \lor C$

b)

b) $(A \lor B) \land (C \lor D)$.

$\equiv (A \land (C \lor D)) \lor (B \land (C \lor D))$ distributivity.

$\equiv ((A \land C) \lor (A \land D)) \lor ((B \land C) \lor (B \land D))$ distributivity.

$\equiv (A \land C) \lor (A \land D) \lor (B \land C) \lor (B \land D)$

c) $\neg((A \lor B) \rightarrow C)$

$\equiv \neg(\neg(A \lor B) \lor C)$ transformation of l'implication

$\equiv (A \vee B) \wedge \neg C$

$\equiv \neg C \wedge (A \vee B)$

$\equiv (\neg C \wedge A) \vee (\neg C \wedge B)$ distributivity.

Exercise n=03

a) $(A \vee B) \rightarrow (C \wedge D)$.

$\equiv \neg(A \vee B) \vee (C \wedge D)$ *transformation of implic ation en disjunction.*

$\equiv (\neg A \wedge \neg B) \vee (C \wedge D)$ *Morgan law.*

$\equiv (\neg A \vee (C \wedge D)) \wedge (\neg B \vee (C \wedge D))$ *distributivity.*

$\equiv ((\neg A \vee C) \wedge (\neg A \vee D)) \wedge ((\neg B \vee C) \wedge (\neg B \vee D))$ *distributivity.*

$\equiv (\neg A \vee C) \wedge (\neg A \vee D) \wedge (\neg B \vee C) \wedge (\neg B \vee D)$

b) $(A \vee (\neg B \wedge (C \vee (\neg D \wedge E))))$

$(A \vee (\neg B \wedge (C \vee (\neg D \wedge E))))$

$\equiv (A \vee (\neg B \wedge ((C \vee \neg D) \wedge (C \vee E))))$ *distributiviy.*

$\equiv ((A \vee \neg B) \wedge (A \vee ((C \vee \neg D) \wedge (C \vee E))))$ *distributivity.*

$\equiv ((A \vee \neg B) \wedge ((A \vee C \vee \neg D) \wedge (A \vee C \vee E)))$ *distributivity.*

$\equiv (A \vee \neg B) \wedge (A \vee C \vee \neg D) \wedge (A \vee C \vee E)$

c) $A \Leftrightarrow (B \wedge \neg C)$.

$\equiv (A \rightarrow (B \wedge \neg C)) \wedge ((B \wedge \neg C) \rightarrow A)$ *transformation of* $\Leftrightarrow$ *in double* $\rightarrow$.

$\equiv (\neg A \vee (B \wedge \neg C)) \wedge (\neg(B \wedge \neg C) \vee A)$ *transformation of implication in disjunction.*

$\equiv (\neg A \vee (B \wedge \neg C)) \wedge ((\neg B \vee C) \vee A)$ *Morgan law .*

$\equiv ((\neg A \vee B) \wedge (\neg A \vee \neg C)) \wedge ((\neg B \vee C) \vee A)$ *distributivity.*

$\equiv (\neg A \vee B) \wedge (\neg A \vee \neg C) \wedge (\neg B \vee C \vee A)$.

d) $A \vee (B \wedge C)$

$(A \lor B) \land (A \lor C)$

e) $(A \land B) \lor C$

$(A \lor C) \land (B \lor C)$

Exercise n=04

1. With the truth table we find F est valid
2. $A \equiv \neg F$; then A is (inconsistency , antilogy,)