

Exercise 1 :

Let be a sinusoidal voltage with an RMS value $U = 15 \text{ V}$ and a period $T = 1 \text{ ms}$.

- 1- Calculate its maximum value, frequency and pulsation.
- 2- Express the instantaneous voltage as a function of time. This voltage is equal to 10 V at the initial instant.
- 3- Determine the complex amplitude of this voltage.

Exercise 2 :

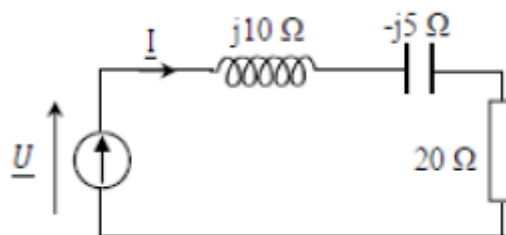
Determine by the complex method, the sum of the three voltages defined by their effective values and their initial phases: $U_1 (55\text{V}, 90^\circ)$, $U_2 (75\text{V}, 45^\circ)$, $U_3 (100\text{V}, 0^\circ)$.

Exercise 3 :

Consider the circuit shown below where $\underline{U}$ is the complex representation of a sinusoidal voltage with an effective value $U_{\text{eff}} = 100 \text{ V}$ and a frequency of 50 Hz .

The components of this circuit are directly characterized by the value of their complex impedance.

1. Calculate the equivalent impedance of the circuit.
2. Calculate the modulus and phase of the equivalent impedance.
3. Calculate the maximum value I_m of the current I ,



Correction 1 :

$$\begin{cases} U_{eff} = 15V \\ T = 1 ms \end{cases}$$

1- Calculate the maximum value of voltage, frequency and pulsation :

$$U_{max} = U_m = U_{eff} \sqrt{2} = 1.41 \times 15 = 21.21 V$$

$$f = \frac{1}{T} = \frac{1}{10^{-3}} = 10^3 Hz = 1 KHz.$$

$$\omega = 2\pi f = 2.3.14.1000 = 6280 rad/s$$

2. Express instantaneous voltage :

The instantaneous value of the voltage is written in the following form :

$$U(t) = U_m \cos(\omega t + \varphi), \varphi = ?,$$

$$U(t=0) = 10 = 21.21 \cos \varphi \Rightarrow \cos \varphi = 10/21.21 = 0.4717$$

$$\Rightarrow \varphi = \arccos(0.4717) = 1.08 \text{ rad.}$$

$$U(t) = 21.21 \cos(6280t + 1.08).$$

3. The complex amplitude of this voltage

$$U(t) \Leftrightarrow \underline{U} = 21.21 e^{j(6280t + 1.08)}$$

Correction 2 :

1. The sum of the three voltages :

$$U_1 (55V, 90^\circ), U_2 (75V, 45^\circ), U_3 (100V, 0^\circ)$$

$\Rightarrow$ Each voltage can be written in the instant form :

$$U(t) = U_m \cos(\omega(0) + \varphi)$$

$\Rightarrow$ It can be associated with a complex number :

$$\underline{U} = U_m e^{j(\omega t + \varphi)} = U_m [\cos(\omega t + \varphi) + j \sin(\omega t + \varphi)]$$

$\Rightarrow$ To facilitate the calculations, we reduce the expression of the complex number :

$$\underline{U} = U_m e^{j\varphi} = U_m [\cos \varphi + j \sin \varphi]$$

$$\underline{U} = U_m e^{j(\varphi)} = U_m [\cos(\varphi) + j \sin(\varphi)]$$

$$U_1 (55 V, 90^\circ) \Leftrightarrow 55 e^{j\frac{\pi}{2}} = 55 (\cos \frac{\pi}{2} + j \sin \frac{\pi}{2}) = 55(0 + j) = 55j$$

$$U_2 (75 V, 45^\circ) \Leftrightarrow 75 e^{j\frac{\pi}{4}} = 75 (\cos \frac{\pi}{4} + j \sin \frac{\pi}{4}) = 75 (\frac{\sqrt{2}}{2} + j \frac{\sqrt{2}}{2}) = 53.03 + j53.03$$

$$U_3 (100 V, 0^\circ) \Leftrightarrow 100 e^{j\frac{\pi}{4}} = 100 (\cos 0 + j \sin 0) = 100(1 + 0) = 100$$

$$\text{The sum of tensions : } \underline{U_{tot}} = \underline{U_1} + \underline{U_2} + \underline{U_3} = 153.03 + j108.03$$

$$|\underline{U_{tot}}| = \sqrt{153.03^2 + 108.03^2} = 187.32,$$

$$\text{The argument is obtained by the relation: } \varphi = \arctan \frac{108.03}{153.03} = 0.615 \text{ rad.}$$

$$U_{tot}(t) = 187.32 \cos(\omega t + 0.615).$$

Correction 3 :

1. The argument is obtained by the relation :

$$Z_{eq} = Z_R + Z_C + Z_L = 20 - j5 + j10 \\ = 20 + j5.$$

$$2. |Z| = \sqrt{20^2 + 5^2} = \sqrt{425} = 20.62$$

$$\varphi = \arctan \frac{5}{20} = \arctan \frac{1}{4} = \arctan 0.25 = 0.25$$

$$\varphi = 0.25.$$

$$3. \underline{I} = \frac{\underline{U}}{\underline{Z}_{eq}}, |\underline{I}| = \frac{|\underline{U}|}{|Z_{eq}|} \rightarrow I_m = \frac{U_m}{|Z_{eq}|} = \frac{U_{eff}\sqrt{2}}{20.62} = \frac{141.42}{20.62} = 6.86 \text{ A}.$$