In practice, between the moment when no current flows and the moment when, experimentally, it is observed that the regime is continuous, there is a period when the currents and voltages evolve over time to reach their final value; This temporary regime is called the "transitional regime".

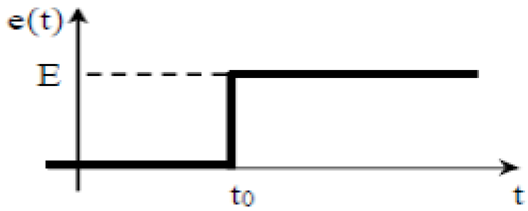
1. **Tension echelon**

   Let be a source of electromotive force voltage (emf) e (t) defined by :

   $$e(t) = \begin{cases} 0\ pour\ t < t_0 \\ 1\ pour\ t > t_0 \end{cases}$$

   with: Constant E

   We say that such a source delivers a voltage step, the graph of this emf is represented on the following figure :



The simplest method to make such a source is to take a voltage source

E continues and a switch in series, which is closed at t = t_0.

"Echelon" or Heaviside function, denoted by H (t), it is defined by :

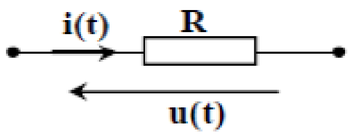
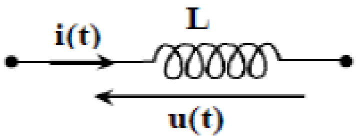
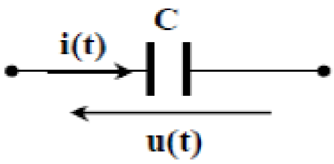
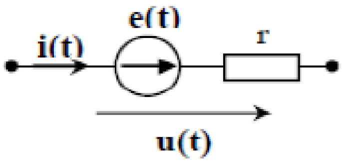
$$H(t) = \begin{cases} 0\ pour\ t < t_0 \\ 1\ pour\ t > t_0 \end{cases}$$

It allows you to represent discontinuous and constant functions in pieces. Thus the voltage step defined at the beginning of this paragraph is written :

$$e(t) = E.H(t)$$

2. Basic Dipoles of Circuits

The voltage-current relationship of a passive or active dipole at a given time is an instantaneous relationship.

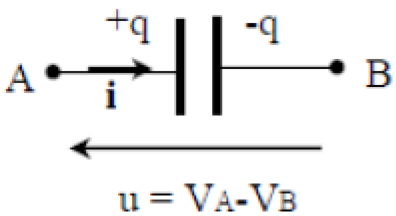
Dipole	Symbol	Voltage-current relationship
Resistor		$u(t) = R\, i(t)$
Inductance		$u(t) = L\frac{di}{dt}$; $i(t) = \frac{1}{L}\int u\, dt$
Capacitor		$i(t) = C\frac{du}{dt}$; $u(t) = \frac{1}{C}\int i\, dt$
Generator		$u(t) = e(t) - r\, i(t)$

3. Study of an RC circuit

3.1.Capacitor Basics

A capacitor consists of two conductive surfaces separated by an insulator (dielectric) that maybe dry air, alumina ... The loads located on these two surfaces are equal in value absolute and opposite signs.

Symbol

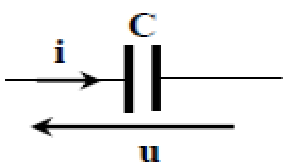


The charge q is related to the potential difference u_{AB} by the relation: $q = u_{AB}.C$.

C: is the capacitance of the capacitor, it is expressed in Farad (F).

Using the receiver convention (i and u are in opposite directions), we obtain the relationship:

$$i(t) = C\frac{du}{dt} = \frac{dq}{dt}$$

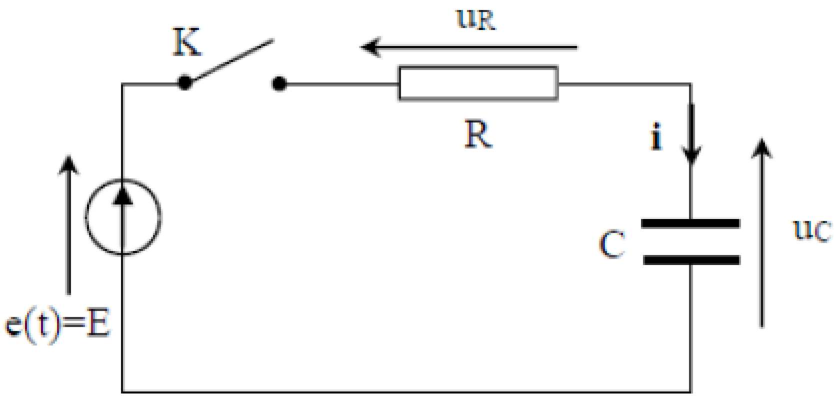


Note, if i and u are of the same direction, then:

$$\boxed{i(t) = C\frac{du}{dt}}$$

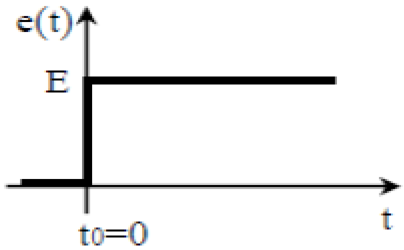
3.2.Response of a circuit (from a dipole) RC to a tension step

An RC dipole is the association in series of a capacitor C and an ohmic conductor of resistance R. The following setup allows us to study the response of an RC circuit to a voltage step. The capacitor is initially discharged.



At time t = 0 , we close the switch K (the capacitor is initially discharged). The capacitor will gradually charge until a voltage opposite to the emf E is established at its terminals.

$$\begin{cases} t < 0 : e(t) = 0 \Rightarrow u_c(t) = 0 \\ t > 0 : e(t) = E = u_c + Ri \end{cases}$$



For t > 0, the law of lattices is written : $E = U_R + U_C = Ri + U_C = RC\frac{dU_c(t)}{dt} + U_C$

Either : $\frac{dU_c}{dt} + \frac{U_c}{RC} = \frac{E}{RC}$

To determine u (t) c when t > 0, we need to solve this differential equation which is a linear differential equation of the first order, with constant coefficients, with second member. His general solution is equal to the sum of the general solution of the equation without a second member (called homogeneous equation) and a particular solution of the equation with second member.

Solving the equation without a second member :.

$$\frac{dU_C}{dt} + \frac{U_C}{RC} = 0 \Leftrightarrow \frac{dU_C}{U_C} = -\frac{dt}{RC}$$

If two expressions are equal, their primitives are equal to one constant :

$$\Leftrightarrow LnU_C = -\frac{1}{RC}dt + const$$

$$\Leftrightarrow U_C(t) = e^{-\frac{1}{RC}t+const} = e^{-\frac{1}{RC}t}.e^{const} = Ae^{-\frac{1}{RC}t}$$

with A: constant to be defined later.

Special solution: obtained when t → ∞, i.e. in steady state.

When t → ∞, all currents and voltages are constant because the generator is constant :

Uc = Const ⇒ i = 0 ⇔ Uc(t) = E.

General solution :

The general solution is equal to the sum of the solution of the equation without a second member and the particular solution : $U_C(t) = Ae^{-\frac{t}{\tau}} + E$

Such as : τ = RC circuit time constant, or relaxation time.

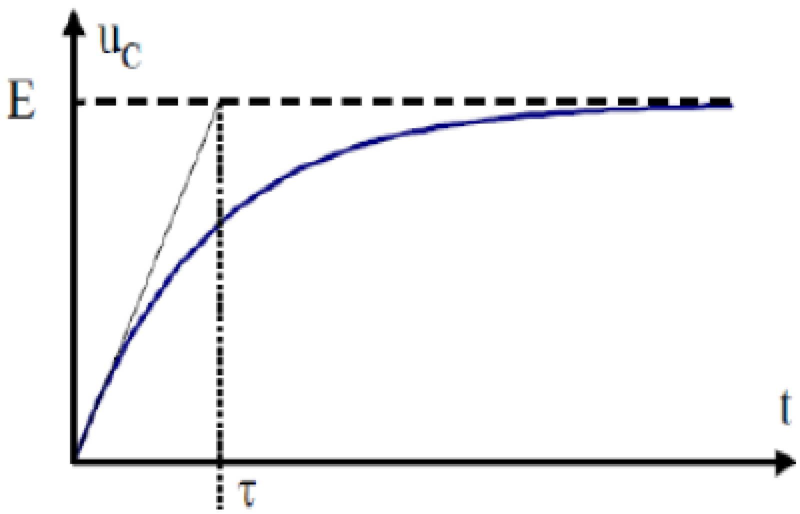
Determination of the constant A by the initial conditions :

The voltage across the capacitor may not have any discontinuities :

$Uc(t = 0^-) = Uc(t = 0^+) = 0$, A = -E, finaly ;

$Uc(t) = -Ee^{-\frac{t}{\tau}} + E = E(-e^{-\frac{t}{\tau}} + 1)$, $U_C(t) = E\left(1 - e^{-\frac{t}{\tau}}\right)$.

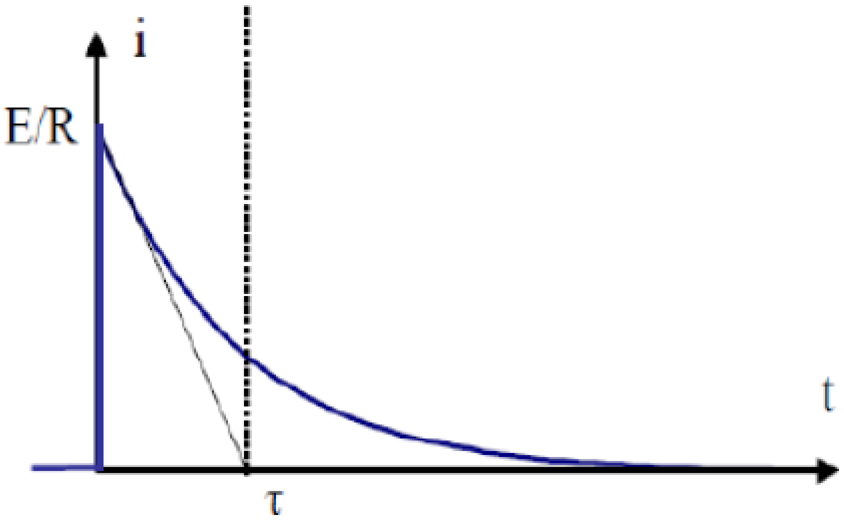
The curve Uc = f(t) is represented as follows :



The charge of the capacitor is not instantaneous, it is a transient phenomenon.

The expression of i(t):

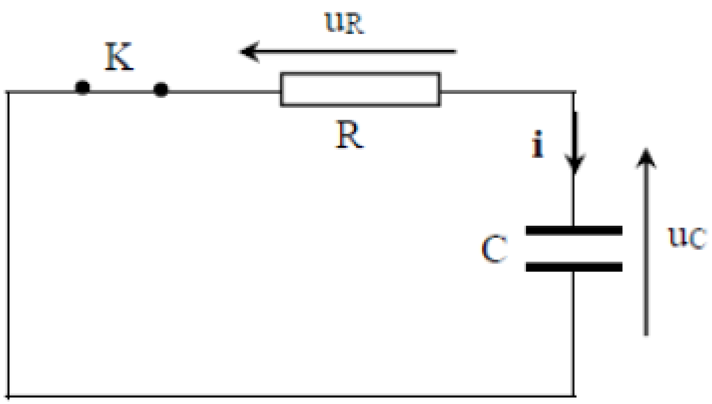
$$i(t) = C\frac{dU_C}{dt} \Rightarrow i(t) = \frac{E}{R}\,e^{-\frac{t}{\tau}}$$



3.2.1. Discharging a capacitor into a resistor

Initially, the capacitor carries the charge Q_0. At time t = 0 we close the switch K.

The capacitor then discharges through the resistor R until its charge is cancelled. The current then becomes zero.



At t = 0, **$U_c = U_{co} = E = Q_0/C$.**

Law of meshes : $U_c + U_R = 0 \Rightarrow U_c + Ri = 0 \Rightarrow$ $$\frac{dU_c}{dt} + \frac{U_C}{RC} = 0$$

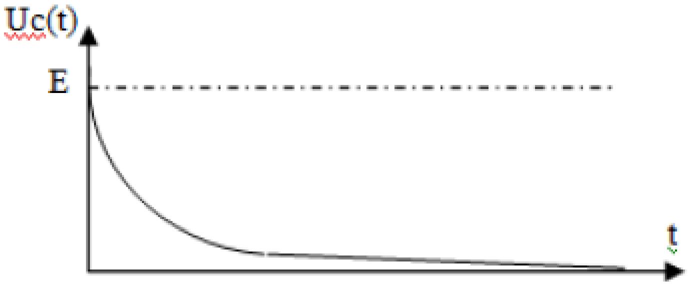
The homogeneity of this equation shows that RC has the dimension of a time. Let us assume τ = RC.

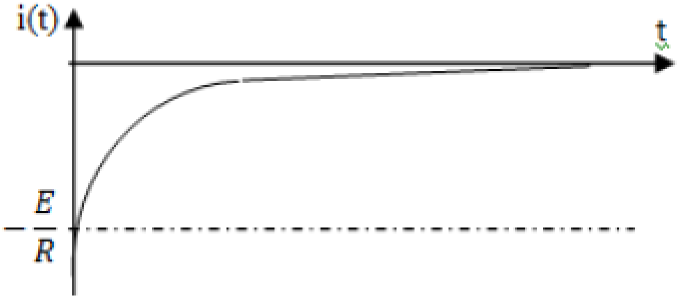
τ is called the time constant of the RC circuit.

The equation is written : $$\frac{dU_C}{dt} + \frac{U_C}{\tau} = 0$$

We obtain a first-order differential equation with constant coefficients, without a second member, whose final solution is :

$$U_C(t) = \frac{Q_0}{C} e^{-\frac{t}{\tau}}, \; i(t) = -\frac{Q_0}{RC} e^{-\frac{t}{\tau}}$$





3.2.2. Electrostatic energy

A capacitor stores energy during charging, it returns this stored energy during discharge. The energy stored by the capacitor for t → ∞ (i.e. at the end of the capacitor charge) is expressed as :

$$\varepsilon_C = \int P(t)dt = \int U(t).i(t).dt = \frac{Q_0^2}{RC^2}\int_0^\infty \exp\left(-\frac{2t}{\tau}\right)dt = \frac{E^2}{R}\int_0^\infty \exp\left(-\frac{2t}{\tau}\right)dt = \frac{E^2}{R}\left[\frac{\exp\left(-\frac{2t}{\tau}\right)}{-2/\tau}\right]_0^\infty$$

$$\varepsilon_C = \frac{1}{2}CE^2 = \frac{Q_0^2}{2C}$$