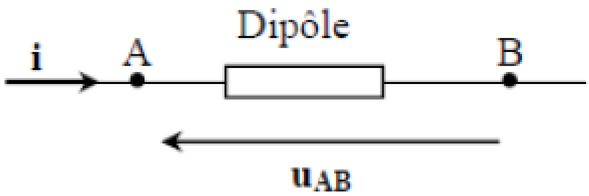
Chap 1 : Electrical networks in continuous operation

A linear circuit is a circuit made up of linear dipoles (resistor, capacitor, coil, voltage and/or current generator). In this chapter, we give simple laws to simply determine the intensity and/or voltage across any dipole in a circuit operating in a DC circuit, knowing the characteristics of the dipoles constituting it.

1. Definitions

1.1. Dipole

A dipole is a circuit accessible by two terminals A and B, it can be characterized by a current i that passes through it and the voltage u, between its terminals.



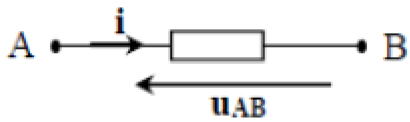
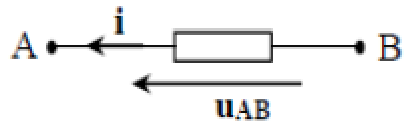
i : electric current flowing from A to B, expressed in Ampere (A).

$u_{AB} = u_A - u_B$: voltage (potential difference) between A and B, expressed in volts (V).

• The characteristic of a dipole is the relation between you and i, it is written in the form u (i).

• The direction of current flow can be: i_{AB} or i_{BA}, with $i_{AB} = - i_{BA}$.

• A dipole can be a receiver or a generator :

Receiver	The current and voltage arrows are in reverse directions.	
Generator	The current and voltage arrows are in the same direction	

1.2. Electrical Regimes

A linear electrical circuit is powered by generators. There are two types of continuous and alternative sources (generators):

• Direct (static) regime: Electrical quantities (voltages and currents) are time-invariant.

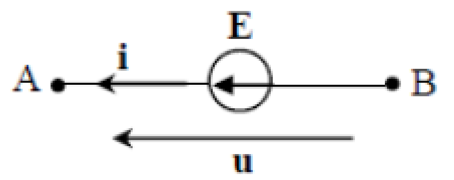
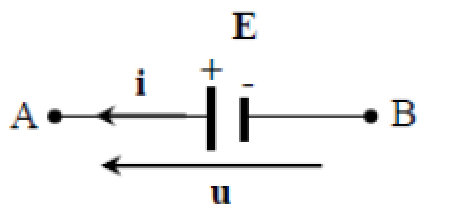
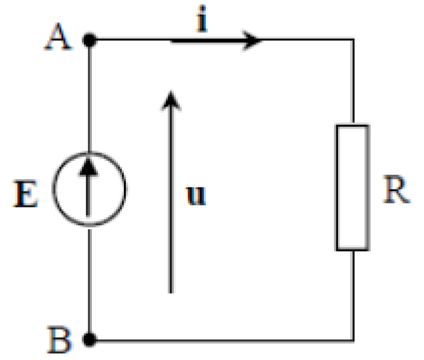
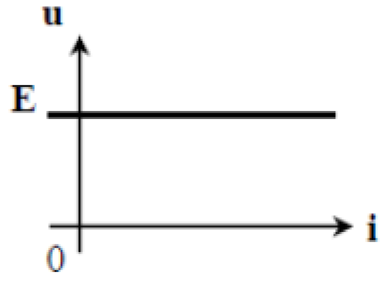
• Variable speed (dynamic): electrical quantities evolve over time, the sources are said to be alternative.

1.3. Direct voltage and current generators

1.3.1. Ideal Voltage Generator

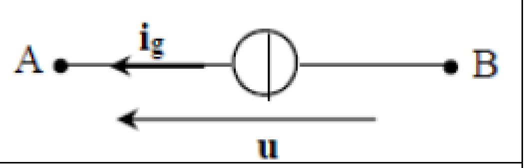
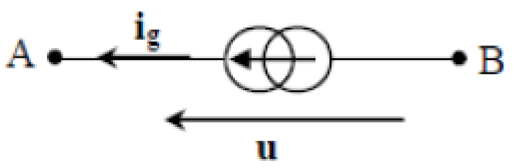
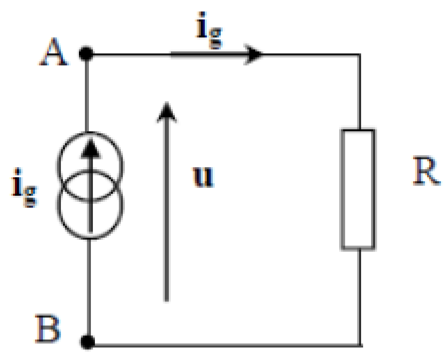
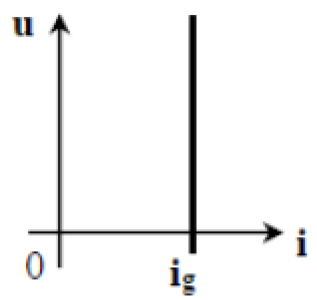
A direct voltage generator (source) is a dipole capable of imposing a constant voltage across its terminals regardless of the intensity of the current flowing through it. Its two representations are :

E: is the electromotive force of the generator (emf).

Representation	In a circuit	Characteristic
 		

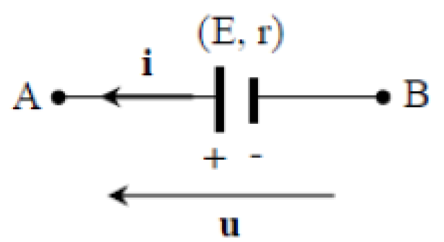
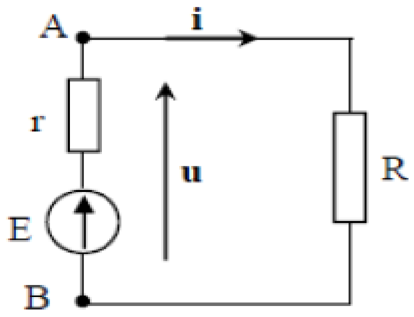
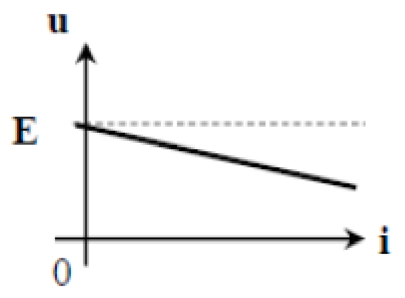
1.3.2. Ideal power generator

Such a generator delivers a current, known as a short-circuit current, independent of the voltage present at its terminals. Its two representations are :

Representation	In a circuit	Characteristic
 		

1.3.3. Real (ohmic) voltage generator

In reality, generators are not perfect and it is considered that a model closer to reality is to associate an ideal voltage generator in series with a resistor. This resistance is called the "internal resistance" of the generator.

Representation	In a circuit	Characteristic
		

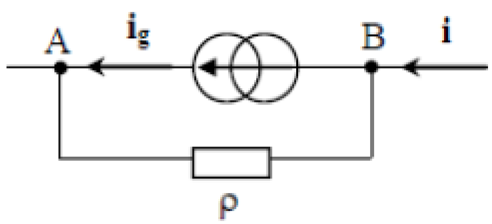
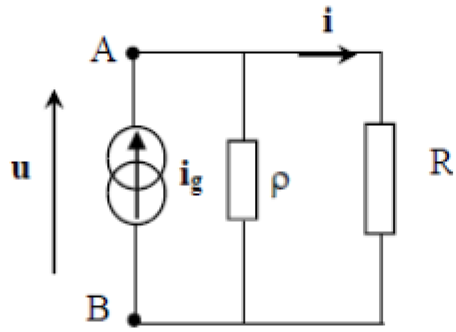
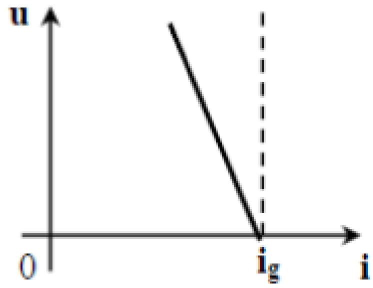
The equation of the characteristic: $u = E - r\,i$.

E : is the electromotive force of the generator (emf).

r : Internal resistance.

1.3.4. Real Current Generator (Ohmic)

In this case, an ideal current generator in parallel is combined with a resistor.

Representation	In a circuit	Characteristic
		

The equation of the characteristic of the real current generator is: $i = i_g - u/\rho$

1.3.5. Electrical power (Adaptation)

The electrical power supplied by a generator (E, r), at a resistive load R, is expressed by :

$P(R) = Ri^2$ avec $i = E/(R+r) \Rightarrow P(R) = E^2.R/(R+r)^2$.

P(R) is maximum, if $\partial P(R)/\partial R = 0$, hence $R = r$ and $P(R)_{max} = E^2/4.r$.

A generator delivers a maximum power in a resistive load (resistance) R, when this is equal to its internal resistance r (R = r). In this case, the generator is said to be suitable for the load.

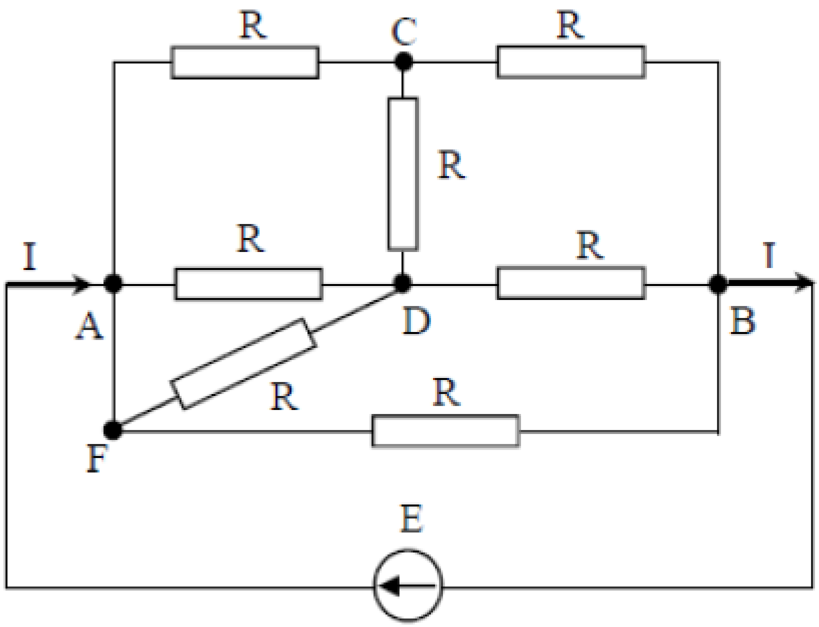
2.1. Linear power grid

A linear electrical network is a combination of passive elements (resistors, capacitors, and inductors) and active elements (voltage and current generators), connected to each other by conductors that are assumed to be resistance-free (perfect).

• A node of a network is a point in the circuit where at least three conductors end (A, B, C..)

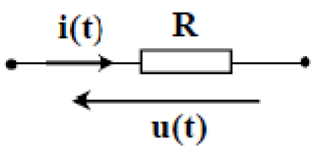
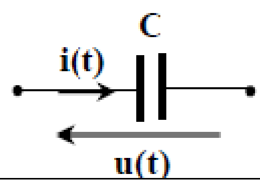
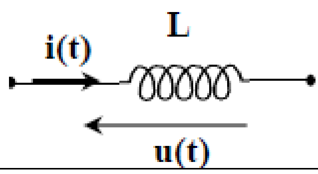
• A branch of the network is a portion of a circuit, located between two consecutive nodes (AC, AD, CB, ...)

• A mesh is a closed loop bounded by branches of the electrical grid (ACDA), (CBDC).

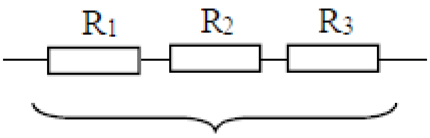
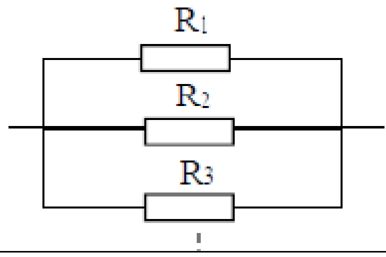
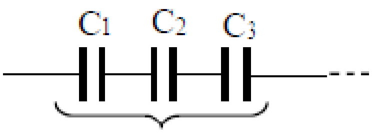
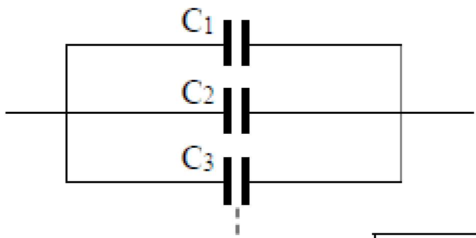


2.2. Linear passive dipoles

Three passive dipoles are commonly used in electrical circuits.

Passive dipole	Basic Law	Representation	In continuous mode
. **Resistor** R(Ω) . **Conductance** G=1/R (S) [Siemens :(Ω^{-1})]	U(t) = R.i(t) I(t) = Gu(t) Ohm's Law		I and u are constant U=RI P(t) = UI=RI^2=U^2/R
.**Capacitor** C ability (F:Farad)	**i(t) = C.du(t)/dt**		u is constant and i is zero The capacitor is an open switch
. **Inductance** L : Coil inductance (H : Henry).	**u(t) = L.di(t)/dt**		i is constant and u is zero . The perfect coil is equivalent to a thread

2.3. Groupement des dipôles passifs

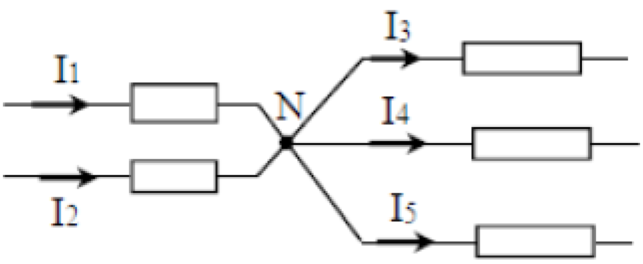
Dipole	**Serial grouping**	**Parallel grouping**
Resistor	 $R_{eq} = R_1 + R_2 + R_3 + ...$	 $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + ...$ $G_{eq} = G_1 + G_2 + G_3 + ...$
Capacitor	 $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + ...$	 $C_{eq} = C_1 + C_2 + C_3 + ...$
inductance	**Inductances follow the same association rules as resistances, provided that there is no coupling between them.**	

2.4. Kirchhoff's Laws

2.4.1. Kirchhoff's Law of Nodes

Kirchhoff's first law is the law of knots: The sum of the intensities of the currents entering a node is equal to the sum of the intensities of the currents leaving it (no charge accumulation).

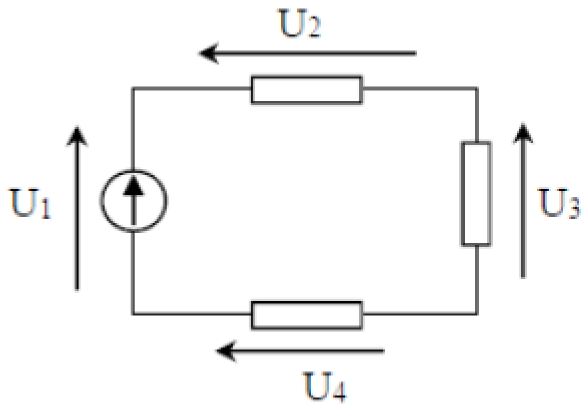
$$I1 + I2 = I3 + I4 + I5$$



2.4.2. Kirchhoff's Law of Mesh

Kirchhoff's second law states: The algebraic sum of potential differences (or tension) along any lattice is zero:

$$U_1 - U_2 - U_3 + U_4 = 0$$

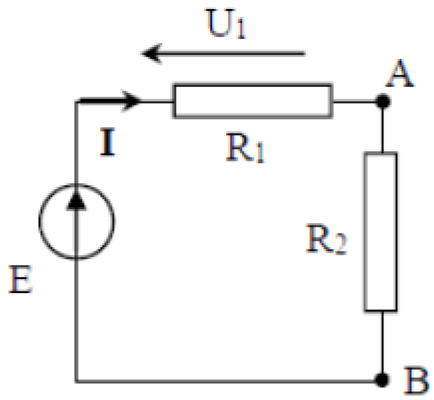


All the voltages Ui are oriented according to the direction of travel on the mesh.

2.5. Fundamental theorems

2.5.1. Voltage divider bridge

• The diagram of a voltage divider bridge is given in the following figure :



• This is a direct application of the series of two resistors :

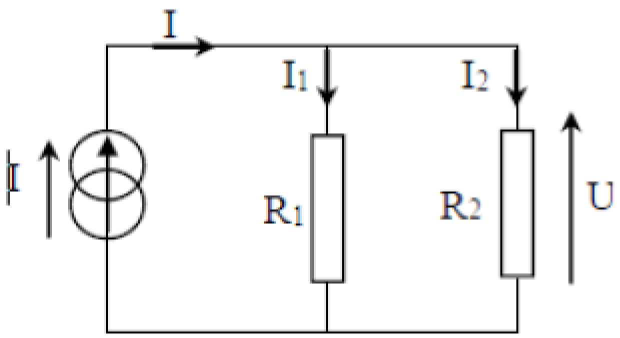
$$E = U1+U2 = I.R1+I.R2, \text{ where from } U2 = (R2/(R1+R2)).E$$

• Generally speaking, the voltage across a resistor placed in a circuit with n resistors in series, powered by a voltage source E is :

$$U_1 = (R_1/(R_1+R_2+R_3+\ldots R_n)).E$$

2.5.2. Current divider bridge

• The diagram of a current divider bridge is given in the following figure (resistors in parallel**) :**



• Let us call U the potential difference that lies at the terminals of the different elements in parallel, we obtain :

$U = R_2.I_2 = ((R_1.R_2)/(R_1+R_2))..I$, where from $I_2 = (R_1/(R_1+R_2)).I$

• If we now divide the numerator and the denominator by the product (R1. R2), we obtain the following relationship :

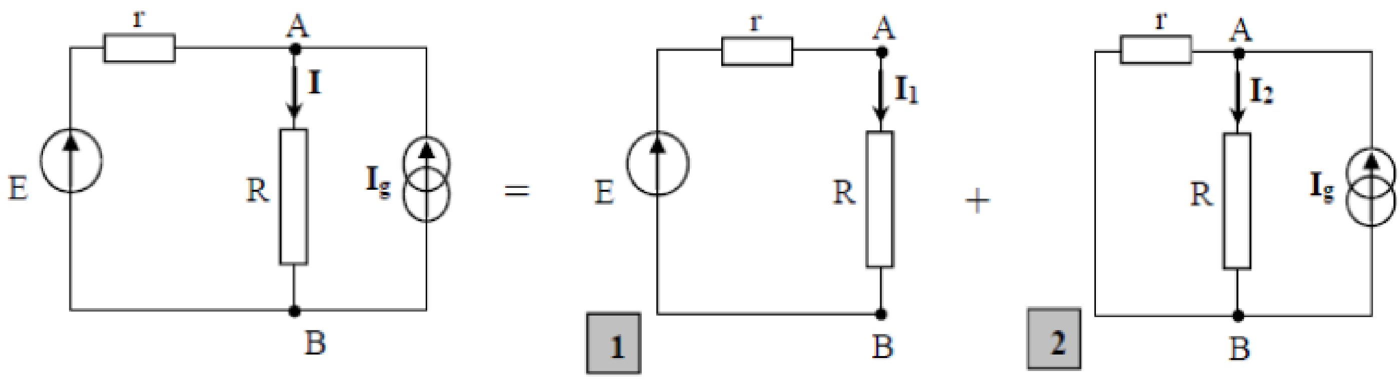
$$I_2 = (G_2/(G_1+G_2)).I$$

• Generally speaking, the current flowing through a resistor Ri placed in a circuit with n resistors in parallel, fed by an ideal source of current I, is :

$$I_i = (G_i/(G_1+G_2+G_3+\ldots\ldots.G_n)).I$$

2.5.3. Superposition theorem

For example, let's take the following figure (circuit powered by two independent sources) :



• assembly 1: the current source Ig being neutralized, the generator (E, r) flows a current I1 into the branch AB of the circuit: ( )

$$I_1 = (E/(R+r))$$

• assembly 2: the generator (E, r) being neutralized (replaced by its internal resistor), the current source activated alone. The current in the resistor R would be I_2:

$$I_2 = (r/(r+R))I_g$$

The current I in the AB branch due to the contribution of the two sources will be: $I = I_1+I_2$

$$I = I_1 + I_2 \ ((E+rIg)/(r+R))$$

2.5.4. Théorèmes de Thévenin et de Norton

2.5.4.1. Théorème de Thévenin

It is possible to replace a portion of the linear electrical network, considered between two terminals A and B, by a voltage generator, known as a "Thévenin generator", with the following characteristics:

- Its internal resistance RTh is the equivalent resistance between terminals A and B when each independent generator is passivated (replaced by its internal resistance).

- Its emf ETh is the voltage measured between A and B at no load (the dipole is not connected to other external elements. Let's take for example the assembly of the following figure:

- The RTh resistance is obtained by passivating the voltage source E:

$$R_{Th} = (R1//R2) = ((R1.R2)/(R1+R2)).$$

-The voltage E_{Th} is the voltage obtained between A and B (voltage across R_2):

$$E_{th} = (R_2=(R_1+R_2)).E.$$

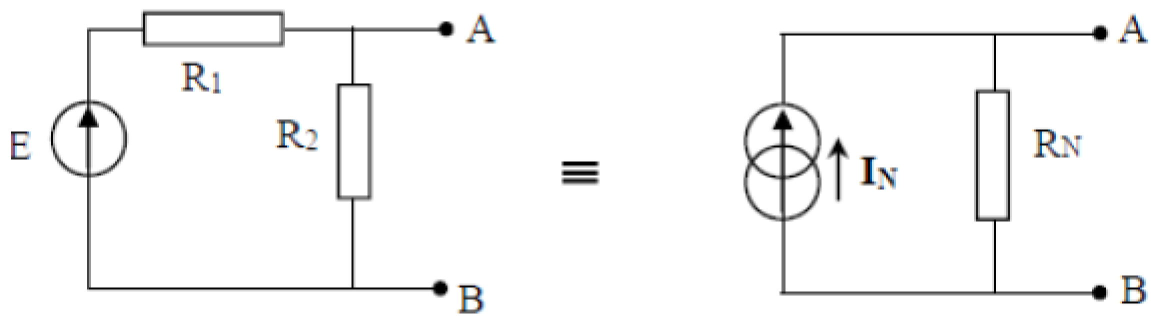
2.5.4.2. Norton's Theorem

It is possible to replace a portion of the electrical network, considered between two terminals A and B, by a current generator, known as a "Norton generator", having the following characteristics:

- Its internal resistance R_N is the Northon resistance.

- Its current I_N is equal to the short-circuit current when points A and B are connected by a wire.

Let's take for example the assembly of the following figure :



- The resistance R_N is obtained by passivating the voltage source E :

$$R_N = ((R_1.R_2)/(R_1+R_2))$$

- The current I_N is the current obtained by shorting the resistor R_2 :

$$I_N = E/R_1$$

Note: Switching from the model of a Thévenin generator to that of a Norton generator leads to finding :

$$R_N = R_{TH},\ I_N = E_{TH}/R_{TH}$$

2.5.5. Association of Voltage Generators in Series

The dipole equivalent to the association in series of n generators of internal resistance voltage r_k and electromotive force E_k is a single voltage generator, including:

- the equivalent resistance is $r_{eq} = \sum_{k=1}^{n} r_k$

- the equivalent electromotive force is $E_{eq} = \sum_{k=1}^{n} E_k$

2.5.6. Parallel Power Generator Association

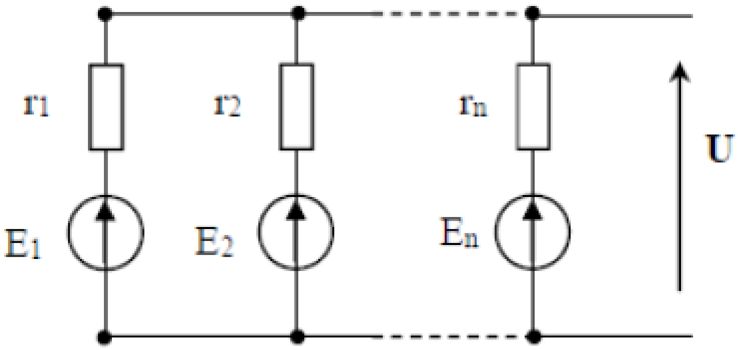
The dipole equivalent to the parallel association of n generators of current generators of internal resistors rk and current Ik is a single current generator, of which :

- the equivalent conductance is : $G_{eq} = \sum_{k=1}^{n} G_k$

- the equivalent current is equal to : $I_{eq} = \sum_{k=1}^{n} I_k$

2.5.7. Millman's theorems

- Millman's theorem, also known as the "knot theorem", allows us to determine the potential of a node where branches composed of a real voltage generator end.



- The proof of this theorem consists in transforming each branch into a current generator :

$$I_i = E_i/r_i = G_iE_i$$

The resulting current $(I = \sum_i I_i)$ circulates in the resistance equivalent to all the resistors in parallel $(I = \sum_i I_i)$. The voltage U is therefore written as $U = \frac{I}{G} = \frac{\sum_{i=1}^{n} G_i E_i}{\sum_{i=1}^{n} G_i}$