Exercise 8 *Let (X, τ) be a topological space, A a topological subspace of X and B a subset of A.*

i) *Show that the closure of B in A is equal to $A \cap \overline{B}$.*

ii) *Show that the set of accumulation points of B in A is equal to $A \cap B'$.*

iii) *Show that the interior of B in A contains the interior of B in X (no equality in general).*

iv) *Show that if A is open in X, then the interior of B in A coincides with the interior of B in X.*

Exercise 9 *Let X be a topological space. Show that an application f from X to $\mathbb{R}$ (equipped with the usual topology) is continuous if and only if $f^{-1}(]a, +\infty[)$ and $f^{-1}(]-\infty, a[)$ are open whatever $a \in \mathbb{R}$.*

Exercise 10 *Let f, g be two continuous applications from X to Y, topological spaces, Y being separated.*

1. *Show that the set $\{x \in X \ ; \ f(x) = g(x)\}$ is closed in X.*
2. *Deduce that if f and g coincide on an everywhere dense subset of X, then $f = g$.*

Exercise 11 *Let X, Y be topological spaces and A, B be non-empty subsets of X such that $X = A \cup B$. Let $f_1 : A \to Y$ and $f_2 : B \to Y$ be two continuous applications such that for all $x \in A \cap B$, we have $f_1(x) = f_2(x)$. We set:*

$$f(x) = \begin{cases} f_1(x) & \text{if } x \in A, \\ f_2(x) & \text{if } x \in B. \end{cases}$$

1. *Let U be a subset of Y. Determine $f^{-1}(U)$ in terms of $f_1^{-1}(U)$ and $f_2^{-1}(U)$.*
2. *Deduce that if A and B are open (resp. closed), then f is continuous.*

Exercise 12 *We take $X =]0, 1[\cup \{2\}$, $Y =]0, 1]$ (each equipped with the topology induced by the usual topology of $\mathbb{R}$) and we consider the application $f : X \to Y$ given by*

$$f(x) = \begin{cases} x & \text{if } 0 < x < 1, \\ 1 & \text{if } x = 2. \end{cases}$$

1. *Verify that f is a bijection and determine its reciprocal f^{-1}.*
2. *Verify that f is continuous on X.*
3. *Show that f^{-1} is not continuous at 1 (We can consider the real sequence with general term $x_n = 1 - \frac{1}{n}$). Conclude.*

Exercise 13 *Let X and Y be two homeomorphic topological spaces.*

1. *Show that X is separated if and only if Y is separated.*
2. *Show that X is separable if and only if Y is separable.*