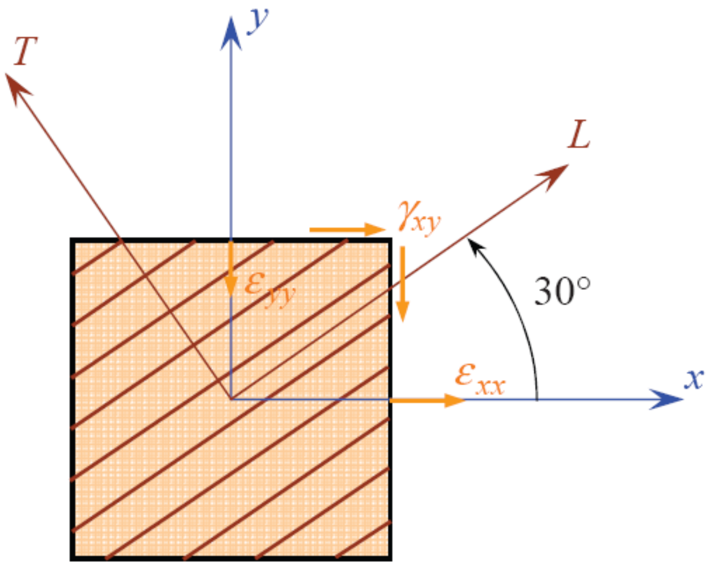
<u>Solution Ex 1</u>



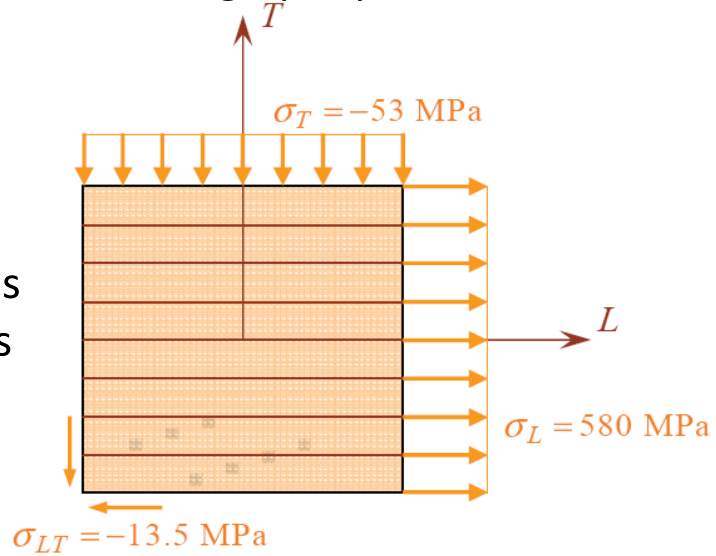
$$\mathbf{Q} = \begin{bmatrix} 41,051 & 3,284 & 0 \\ 3,284 & 10,263 & 0 \\ 0 & 0 & 4,5 \end{bmatrix} \text{GPa} \qquad \bar{Q} = \begin{bmatrix} 28.339 & 8.299 & 9.561 \\ 8.299 & 12.945 & 3.770 \\ 9.561 & 3.770 & 9.515 \end{bmatrix} \text{GPa}$$

$$T = \begin{bmatrix} \frac{3}{4} & \frac{1}{4} & \frac{1}{2}\sqrt{3} \\ \frac{1}{4} & \frac{3}{4} & -\frac{1}{2}\sqrt{3} \\ -\frac{1}{4}\sqrt{3} & \frac{1}{4}\sqrt{3} & \frac{1}{2} \end{bmatrix}$$

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} 28.339 & 8.299 & 9.561 \\ 8.299 & 12.945 & 3.770 \\ 9.561 & 3.770 & 9.515 \end{bmatrix} \times 10^{-9} \begin{bmatrix} 10 \\ -5 \\ 20 \end{bmatrix} \times 10^{-3}$$

$$\sigma_x = 433 MPa$$
$$\sigma_y = 94 MPa$$
$$\tau_{xy} = 267 MPa$$

Représentation graphique des résultats



$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} = T \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{Bmatrix} 580 \\ -53 \\ -13.5 \end{Bmatrix} MPa$$

Contraintes dans les axes naturels

Solution Ex 3

$$G_{12} = \frac{\tau_{12}}{\gamma_{12}}$$

$$\tau_{12} \leftarrow \{\sigma_{(1,2)}\} = [T(\theta)]\{\sigma_{(x,y)}\}$$

$$\gamma_{12} \leftarrow \{\varepsilon_{(1,2)}\} = [T'(\theta)]\{\varepsilon_{(x,y)}\}$$

$$\text{État de contrainte} \begin{Bmatrix} \sigma_x \\ \sigma_y = 0 \\ \tau_{xy} = 0 \end{Bmatrix}$$

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix} = [T'(\theta)] \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{bmatrix} c^2 & s^2 & cs \\ s^2 & c^2 & -cs \\ -2cs & 2cs & (c^2 - s^2) \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}$$

$$\gamma_{12} = -2\cos\theta\sin\theta\,\varepsilon_x + 2\cos\theta\sin\theta\,\varepsilon_y + (\cos^2\theta - \sin^2\theta)\gamma_{xy}$$

$$\gamma_{12} = -\varepsilon_x + \varepsilon_y$$

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} = [T(\theta)] \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} c^2 & s^2 & 2cs \\ s^2 & c^2 & -2cs \\ -cs & cs & (c^2 - s^2) \end{bmatrix} \begin{Bmatrix} \sigma_x \\ 0 \\ 0 \end{Bmatrix}$$

$$\tau_{12} = -\cos\theta\sin\theta\,\sigma_x = -\frac{1}{2}\sigma_x$$

$$G_{12} = \frac{\tau_{12}}{\gamma_{12}} = \frac{-\left(\frac{1}{2}\right)\sigma_x}{-\varepsilon_x + \varepsilon_y} = \frac{\sigma_x}{2(\varepsilon_x - \varepsilon_y)}$$

Solution Ex 5

1. $\{\sigma_{(1,2)}\} = [T(\theta)]\{\sigma_{(x,y)}\}$

$$\begin{Bmatrix}\sigma_1\\ \sigma_2\\ \tau_{12}\end{Bmatrix} = \begin{bmatrix} c^2 & s^2 & 2cs \\ s^2 & c^2 & -2cs \\ -cs & cs & (c^2-s^2) \end{bmatrix} \begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix}$$

$$= \begin{bmatrix} 0.75 & 0.25 & 0.866 \\ 0.25 & 0.75 & -0.866 \\ 0.433 & 0.433 & 0.5 \end{bmatrix} \begin{Bmatrix}100\\ -50\\ 50\end{Bmatrix} = \begin{Bmatrix}105.8\\ -55.8\\ -39.95\end{Bmatrix} \text{MPa}$$

2. $\{\varepsilon_{(1,2)}\} = [S]\{\sigma_{(1,2)}\}$

$$\begin{Bmatrix}\varepsilon_1\\ \varepsilon_2\\ \gamma_{12}\end{Bmatrix} = \begin{bmatrix} \frac{1}{E_1} & -\frac{\nu_{12}}{E_1} & 0 \\ -\frac{\nu_{12}}{E_1} & \frac{1}{E_1} & 0 \\ 0 & 0 & \frac{1}{G_{12}} \end{bmatrix} \begin{Bmatrix}\sigma_1\\ \sigma_2\\ \tau_{12}\end{Bmatrix}$$

$$= \begin{bmatrix} \frac{1}{70} & -\frac{0.25}{70} & 0 \\ -\frac{0.25}{70} & \frac{1}{70} & 0 \\ 0 & 0 & \frac{1}{5} \end{bmatrix} \begin{bmatrix}105.8\\ -55.8\\ -39.95\end{bmatrix} \times 10^{-3} = \begin{bmatrix}0.00171\\ -0.00117\\ -0.808\end{bmatrix}$$

Solution Ex 7 : suite ex 1

On démontre facilement que la loi de Hook-Duhamel s'écrit dans le système de coordonnées x - y comme suite :

$$\{\varepsilon\}_{(x,y)} = [\bar{S}]\{\sigma\}_{(x,y)} + \Delta T\{\alpha\}_{(x,y)}$$

Avec

$$\{\alpha\}_{(x,y)} = \begin{Bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{Bmatrix} \quad \text{tel que :} \quad \begin{aligned} \alpha_x &= \alpha_1 \cos^2\theta + \alpha_2 \sin^2\theta \\ \alpha_y &= \alpha_1 \sin^2\theta + \alpha_2 \cos^2\theta \\ \alpha_y &= 2\sin\theta\cos\theta(\alpha_2 - \alpha_1) \end{aligned}$$

A.N. :

$$[\bar{S}] = [\bar{Q}]^{-1} = \begin{bmatrix} 0.0590 & -0.0232 & -0.0501 \\ -0.0232 & 0.0965 & -0.0149 \\ -0.0501 & -0.0149 & 0.1613 \end{bmatrix} GPa^{-1} \qquad \{\sigma\}_{(x,y)} = \begin{Bmatrix} 580 \\ -53 \\ -13.5 \end{Bmatrix} MPa = \begin{Bmatrix} 580 \\ -53 \\ -13.5 \end{Bmatrix} \times 10^{-3} GPa$$

$$\begin{Bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{Bmatrix} = 10^{-4} \times \begin{Bmatrix} 0.1769 \\ 0.0531 \\ -0.0396 \end{Bmatrix} \frac{1}{°C}$$

On trouve

$$\varepsilon_x = 0.0382$$
$$\varepsilon_y = -0.0178$$
$$\gamma_{xy} = -0.0309$$