

Exercise 01 : Solve the following problems

$$\text{a) By the characteristics : } (E_1) : \begin{cases} y u_x - u_y - 2u = 0 \\ u(0, y) = y \end{cases}, \quad (E_2) : \begin{cases} u_x - x^2 u_y = 0 \\ u(x, 0) = x^3 \end{cases}$$

$$\text{b) By separation of variables : } (E_1) : \begin{cases} 2x u_x + u_y = 0 \\ u(1, 0) = 1 \end{cases}, \quad (E_2) : \begin{cases} u_x + \sin(x) u_y = 0 \\ u\left(\frac{\pi}{2}, 0\right) = -2 \end{cases}$$

$$\text{c) By the change of coordinates : } (E_1) : \begin{cases} u_x + u_y = 1 \\ u(2x, x) = x \end{cases}, \quad (E_2) : \begin{cases} u_x - 3 u_y = 0 \\ u(0, y) = \sin(y) \end{cases}$$

Exercise 02 :

$$\text{Solve : } (E_1) : \begin{cases} u_{xy} = x e^y \\ u(x, 0) = 3x^2 + 5 \\ u(0, y) = -y^3 + 5 \end{cases}, \quad (E_2) : \quad u_{xx} + \frac{1}{x} u_x = 0$$

Exercise 03 :

By the characteristics method solve the following PDE :

$$(E_1) : \quad u_{xx} - 4 u_{yy} = y^2, \quad (E_2) : \begin{cases} u_{xx} - 6 u_{xy} + 9 u_{yy} = x y^2 \\ u(0, y) = 0 \\ u(x, 0) = 0 \end{cases}$$

Exercise 04 : Determine if the following series converges or diverges :

$$\sum_{n \geq 1} \frac{n^n}{2^{n n!}} \quad \sum_{n \geq 1} n \ln \left(1 + \frac{1}{n}\right) \quad \sum_{n \geq 0} \left(\frac{n}{n+1}\right)^{n^2} \quad \sum_{n \geq 1} \left(\frac{2}{3}\right)^{n+1}$$

$$\sum_{n \geq 0} \left(\frac{n}{2n+1}\right)^n \quad \sum_{n \geq 0} (\sqrt{n^3 - 1} - \sqrt{n^3}) \quad \sum_{n \geq 1} \frac{2n^2}{n(n+2)}$$

Exercise 05 : The following series are convergent or divergent ?

$$\sum_{n \geq 1} \cos\left(\frac{1}{n}\right) \quad \sum_{n \geq 0} \sin\left(\frac{4}{3^n}\right) \quad \sum_{n \geq 1} \left(1 - \cos\left(\frac{1}{n}\right)\right) \quad \sum_{n \geq 1} (-1)^n \sin\left(\frac{1}{n^2}\right)$$