Test n°01

Nom & Prénom : **Groupe :**

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1) Compute the sum : $S = \sum_{k \geq 1} \frac{3}{\sqrt{n(2k+n)}}$

2) Consider $D = \{(x, y) \in \mathbb{R}^2 \quad / \ x \geq 0, \quad y \leq x, \quad x^2 + y^2 \leq 4\}$.

   Plot the graph of D. And calculate $\iint_D (x^2 - y^2)dxdy$.

Answer:

Answer: **Test n°01 Maths 03**

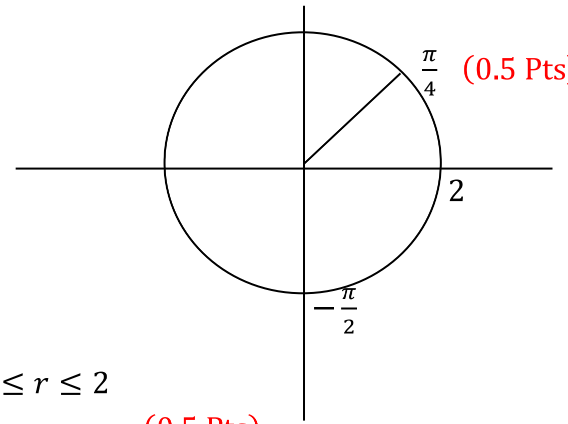
1) $S = \lim_{n \to +\infty} S_n$, $S_n = \sum_{k=1}^{n} \frac{3}{\sqrt{n(2k+n)}} = \sum_{k=1}^{n} \frac{3}{\sqrt{n^2\left(2\frac{k}{n}+1\right)}} = \frac{1}{n}\sum_{k=1}^{n} \frac{3}{\sqrt{2\frac{k}{n}+1}}$ (0.5 Pts)

$$= \frac{b-a}{n}\sum_{k=1}^{n} f\left(a + \frac{b-a}{n}k\right)$$ (0.5 Pts)

$$\begin{cases} b-a=1 \\ f\left(a+\frac{k}{n}\right) = \frac{3}{\sqrt{2\frac{k}{n}+1}} \end{cases} \Longrightarrow \begin{cases} [a \quad b] = [0 \quad 1] \\ f(x) = \frac{3}{\sqrt{2x+1}} \end{cases}$$ (0.5 Pts) , f is continuous on $[0 \quad 1]$, Then :

$S = \int_0^1 f(x)dx$ (0.5 Pts) , $S = \int_0^1 \frac{3}{\sqrt{2x+1}}dx = 3\left(\sqrt{2x+1}\right)_0^1 = 3\left(\sqrt{3}-1\right)$. (0.5 Pts + 0.5 Pts)

(0.5 Pts)



(0.5 Pts)

2) Put the polar coordinates : $\begin{cases} x = r\cos(\theta) \\ y = r\sin(\theta) \end{cases}$, $J = r$

$$D : \begin{cases} x^2+y^2 \le 4 \\ x \ge 0 \\ y \le x \end{cases} \Longrightarrow \begin{cases} x^2+y^2 \le 4 \\ \cos(\theta) \ge 0 \\ \sin(\theta) \le \cos(\theta) \end{cases} \Longrightarrow \begin{cases} 0 \le r \le 2 \\ -\frac{\pi}{2} \le \theta \le \frac{\pi}{4} \end{cases}$$ (0.5 Pts)

$$\iint_D (x^2-y^2)dxdy = \int_0^2 \int_{-\frac{\pi}{2}}^{\frac{\pi}{4}} r^2\left(cos^2(\theta) - sin^2(\theta)\right)|J|drd\theta$$ (0.5 Pts)

$$= \left(\int_0^2 r^3 dr\right)\left(\int_{-\frac{\pi}{2}}^{\frac{\pi}{4}} \cos(2\theta)\, d\theta\right) = \left(\frac{1}{4}r^4\right)_0^2 \left(\frac{1}{2}\sin(2\theta)\right)_{-\frac{\pi}{2}}^{\frac{\pi}{4}} = 2\,.$$ (0.5 Pts+0.5 Pts)

$$\lim_{n\to+\infty} S_n \quad , \quad S_n = \sum_{k=1}^{n} \frac{3}{\sqrt{n(2k+n)}} = \sum_{k=1}^{n} \frac{3}{\sqrt{n^2\left(2\frac{k}{n}+1\right)}} = \frac{3}{n}\sum_{k=1}^{n} \frac{1}{\sqrt{2\frac{k}{n}+1}} \qquad \text{(0.5 Pts)}$$

$$= \frac{b-a}{n}\sum_{k=1}^{n} f\left(a+\frac{b-a}{n}k\right) \qquad \text{(0.5 Pts)}$$

$$\begin{cases} b-a=3 \\ f\left(a+3\frac{k}{n}\right) = \frac{1}{\sqrt{2\frac{k}{n}+1}} \end{cases} \Rightarrow \begin{cases} [a \quad b] = [0 \quad 3] \\ f(x) = \frac{\sqrt{3}}{\sqrt{2x+3}} \end{cases} \qquad \text{(0.5 Pts)}$$

f is continuous on $[0 \quad 3]$, then $S = \int_0^3 f(x)dx$ (0.5 Pts)

$$S = \int_0^1 \frac{\sqrt{3}}{\sqrt{2x+3}}dx = \sqrt{3}\left(\sqrt{2x+3}\right)_0^3 = \sqrt{3}\left(3-\sqrt{3}\right) = 3\left(\sqrt{3}-1\right). \qquad \text{(0.5 Pts+0.5 Pts)}$$