Test n°01

Nom & Prénom : **Groupe :**

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1) Compute the sum : $S = \sum_{k \geq 0} \frac{3k}{n(2k+n)}$

2) Consider D the domain delimited by the vertices : $O(0,0)$, $A(2,2)$, and $B(2,-2)$.

   Plot the graph of D. And calculate $\iint_D (2x - y)dxdy$.

Answer:

Answer: **Test n°01 Maths 03**

1) $S = \lim_{n\to+\infty} S_n$, $S_n = \sum_{k=1}^{n} \frac{3k}{n(2k+n)} = \frac{1}{n}\sum_{k=1}^{n} \frac{3\frac{k}{n}}{2\frac{k}{n}+1}$ (0.5Pts)

$$= \frac{b-a}{n}\sum_{k=1}^{n} f\left(a + \frac{b-a}{n}k\right)$$ (0.5Pts)

$$\begin{cases} b-a=1 \\ f\left(a+\frac{k}{n}\right) = \frac{3\frac{k}{n}}{2\frac{k}{n}+1} \end{cases} \Rightarrow \begin{cases} [a \quad b] = [0 \quad 1] \\ f\left(\frac{k}{n}\right) = \frac{3\frac{k}{n}}{2\frac{k}{n}+1} \ ; f(x) = \frac{3x}{2x+1} \end{cases}$$ (0.5Pts)

f is continuous on$[0 \quad 1]$, then $S = \int_0^1 f(x)dx$. (0.5Pts)

$$S = 3\int_0^1 \frac{x}{2x+1}dx = \frac{3}{2}\int_0^1 \left(1 - \frac{1}{2x+1}\right)dx = \frac{3}{2}\left(x - \frac{1}{2}ln(2x+1)\right)_0^1 = \frac{3}{2}\left(1 - \frac{\ln(3)}{2}\right).$$

(0.5Pts) (0.5Pts)

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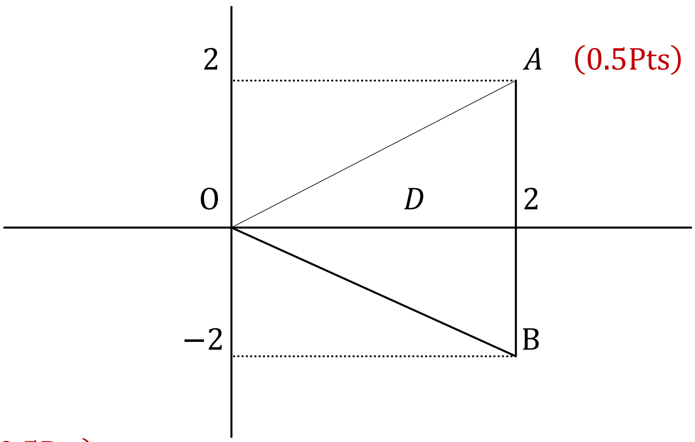
2) D : O(0,0), $A(2,2)$, B(2,−2).



$0 \le x \le 2$, $-x \le y \le x$ (0.5Pts)

$$\iint_D (2x-y)dxdy = \int_0^2 \left(\int_{-x}^{x} (2x-y)dy \right) dx$$ (0.5Pts)

$$= \int_0^2 \left(2xy - \frac{1}{2}y^2\right)_{-x}^{x} dx = \int_0^2 4x^2 dx = \left(\frac{4}{3}x^3\right)_0^2 = \frac{32}{3}.$$ (0.5Pts)

(0.5Pts) (0.5Pts)

$$S = \lim_{n\to+\infty} S_n \quad , \quad S_n = \sum_{k=1}^{n} \frac{3k}{n(2k+n)} = \frac{3}{n}\sum_{k=1}^{n} \frac{\frac{k}{n}}{2\frac{k}{n}+1}$$ (0.5Pts)

$$= \frac{b-a}{n}\sum_{k=1}^{n} f\left(a + \frac{b-a}{n}k\right)$$ (0.5Pts)

$$\begin{cases} b-a=3 \\ f\left(a+3\frac{k}{n}\right) = \dfrac{\frac{k}{n}}{2\frac{k}{n}+1} \end{cases} \Longrightarrow \begin{cases} [a \quad b] = [0 \quad 3] \\ f\left(3\frac{k}{n}\right) = \dfrac{\frac{k}{n}}{2\frac{k}{n}+1} \; ; f(x) = \dfrac{x}{2x+3} \end{cases}$$ (0.5Pts)

f is continuous on $[0 \quad 3]$, then $S = \int_0^3 f(x)dx$ (0.5Pts)

$$S = \int_0^3 \frac{x}{2x+3}dx = \frac{1}{2}\int_0^3 \left(1 - \frac{3}{2x+3}\right)dx$$ (0.5Pts)

$$S = \frac{1}{2}\left(x - \frac{3}{2}ln(2x+3)\right)_0^3 = \frac{3}{2}\left(1 - \frac{\ln(3)}{2}\right).$$ (0.5Pts)