Larbi Ben M’hidi University

Faculty of Exact Sciences, Natural Sciences and Life Sciences

Department of Mathematics and Computer Science

Physics 1

Mechanics of the material point

Dr. Souheyla Gagui

souheyla.gagui@univ-oeb.dz

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Mechanics of the material point

Chapter 1: Kinematics of a material point

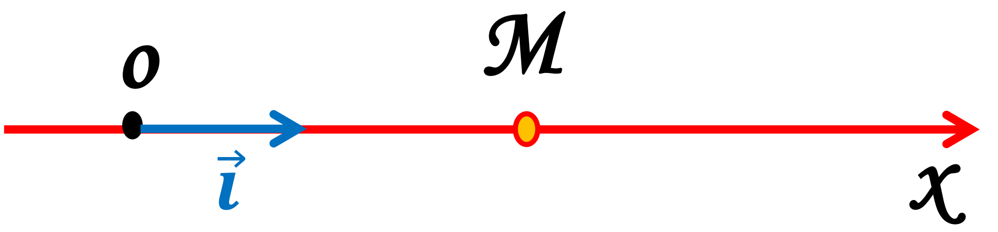
- Movement characteristics
- Rectilinear motion
- Motion in a Plane
- Movement in space
- Relative motion

Rectilinear motion

1. Uniform Rectilinear Motion
2. Uniformly Varied Rectilinear Motion
3. Rectilinear motion with variable acceleration
4. Sinusoidal rectilinear movement

Rectilinear motion

Rectilinear motion is a motion of a material point along a trajectory is a straight line.



Position vector :

$$\vec{r} = \overrightarrow{OM} = x\vec{i}$$

Velocity vector :

$$\vec{V} = V_x\vec{i} = \dot{x}\vec{i}$$

Acceleration vector :

$$\vec{a} = a_x\vec{i} = \ddot{x}\vec{i}$$

1. Uniform rectilinear motion

It is a Uniform rectilinear motion that has *zero acceleration* $(\vec{a} = 0)$ *and a* *constant velocity* $(\vec{v} = C)$.

▪ Time equation of Uniform rectilinear motion

We choose the OX axis as a rectilinear reference and we put the initial condition :

x=f(t) ?

at $t=0\ s \rightarrow x=x_0$

We have : $v = \dfrac{dx}{dt}$

To obtain the expression of the x as a function of time, it is necessary to integrate the expression for the velocity.

$$\Rightarrow dx = vdt \Rightarrow \int_{x_0}^{x} dx = \int_{t_0}^{t} vdt \Rightarrow x\Big|_{x_0}^{x} = vt\Big|_{0}^{t}$$

$$x - x_0 = v\,(t - 0)$$

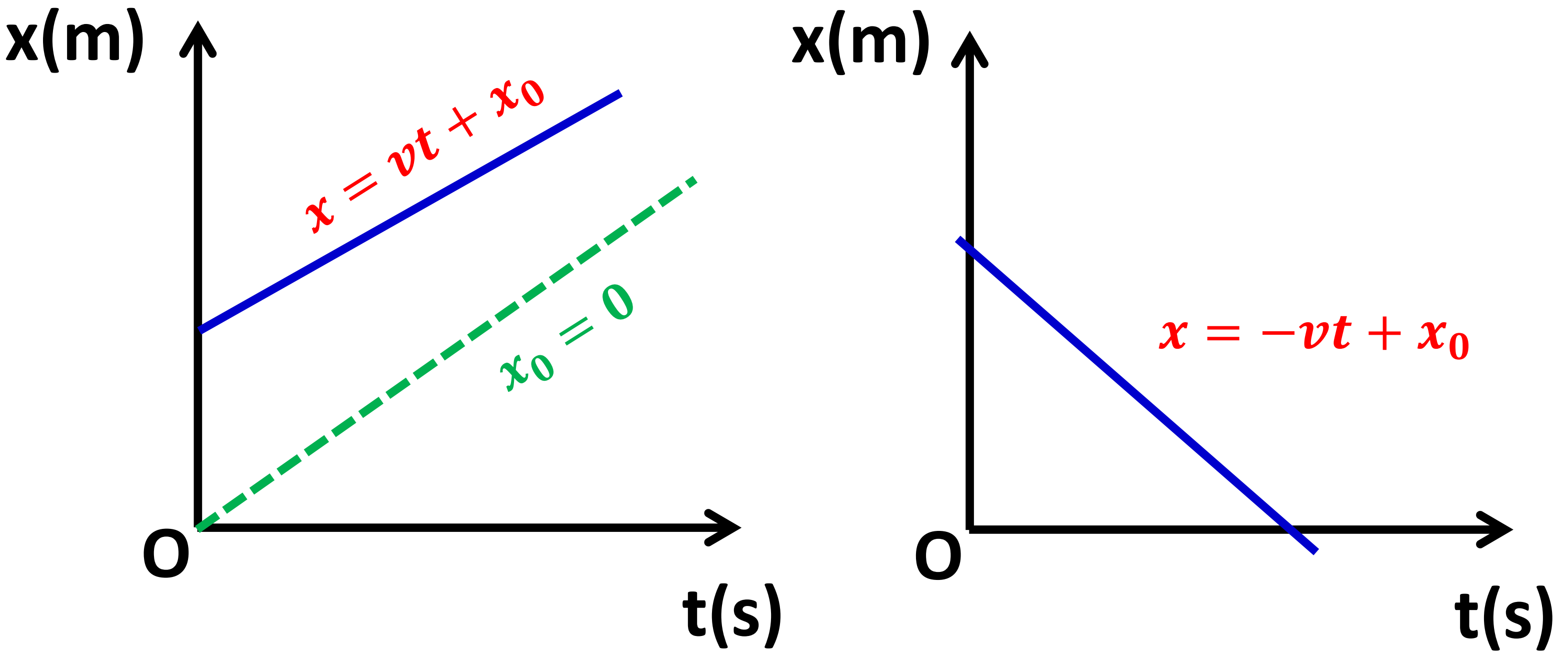
$\Downarrow$

Equation Uniform rectilinear motion

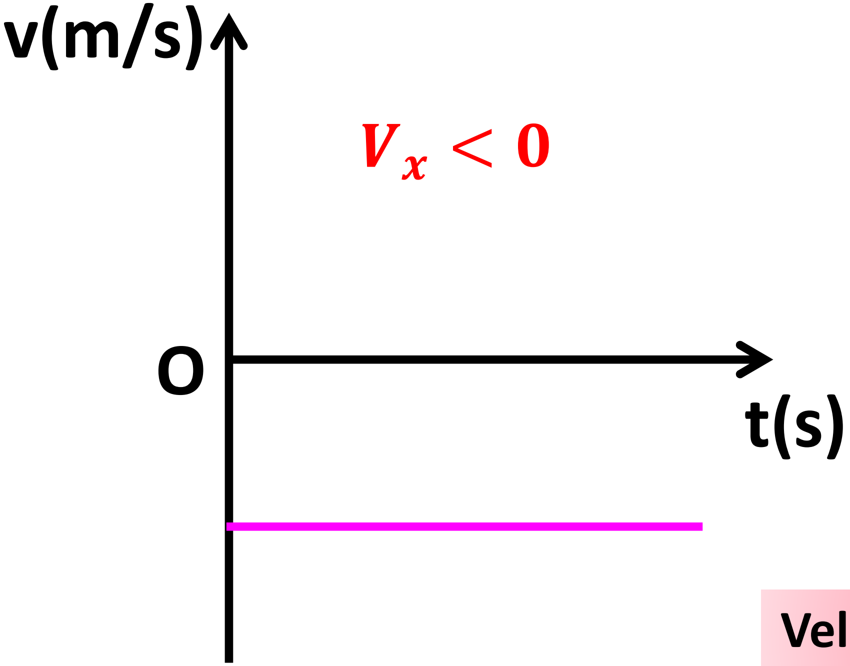
$$x = vt + x_0$$

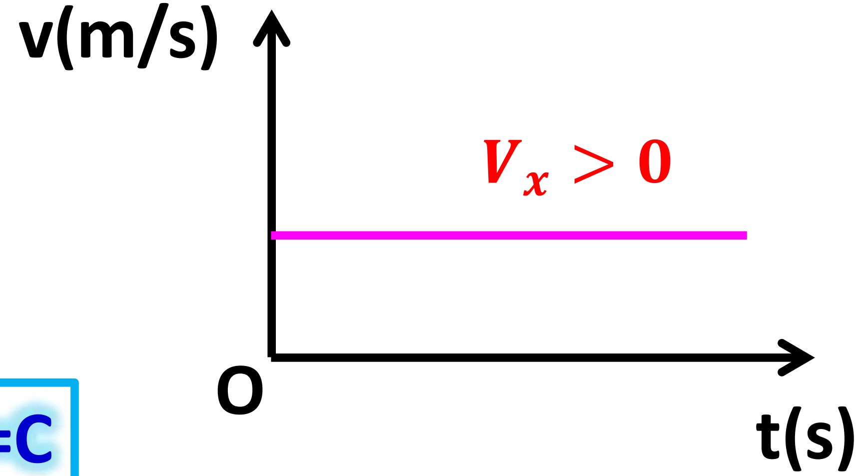
- x: Final position (m) at t
- v: velocity (m/s)
- x_0: Initial position (m) at t=0

The graphical representation of displacement, velocity and acceleration as a function of time for ***uniform rectilinear motion as follows :***



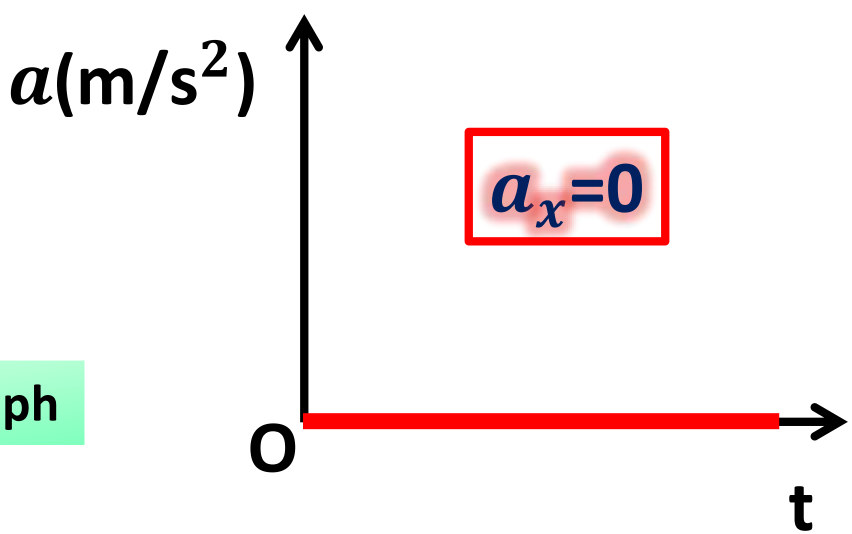
Position – time graph





V_x=C

Velocity – time graph



Acceleration – time graph

Example 1

The time equations of the motion of a material point are :

$x = \frac{t}{3} + 1\,; \quad y = 3t + 6\,; \quad z = 0$ *(all units are in the international system).*

- *Show that the motion is rectilinear and uniform*

Solution

- *Let's first prove the motion is rectilinear by finding the trajectory equation.*

$$\begin{cases} x = \frac{t}{3} + 1 \\ y = 3t + 6 \end{cases} \Rightarrow \begin{cases} x - 1 = \frac{t}{3} \\ y = 3t + 6 \end{cases} \Rightarrow \begin{cases} t = 3(x-1) & (1) \\ y = 3t + 6 & (2) \end{cases}$$

$(1) \to (2) \Rightarrow y = 3 \times 3(x-1) + 6 \Rightarrow y = 9x - 9 + 6$

$$y = 9x - 3$$

Equation of a straight line, so the movement is rectilinear.

- *For this movement to be uniform, the Velocity must be constant.*

We have the position vector is : $\vec{r} = \overrightarrow{OM} = \left(\frac{t}{3} + 1\right)\vec{i} + (3t+6)\vec{j} + 0\vec{k}$

The velocity vector is : $\vec{v} = \frac{d\overrightarrow{OM}}{dt} = \frac{1}{3}\vec{i} + 3\vec{j}$

$$\Downarrow$$

$$v = \sqrt{\left(\frac{1}{3}\right)^2 + 3^2} = 3.01\ (m/s) = C$$

v= C $\Rightarrow$ *This indicates that the movement is uniform.*

The movement rectilinear and uniform.

2. Uniformly Varied Rectilinear Motion

It is a rectilinear motion with constant acceleration $(\vec{a} = C)$

- Time equation of Uniformly Varied Rectilinear Motion

We put the initial condition : at $t=0\ s \rightarrow v=v_0$

We have : $a = \dfrac{dv}{dt} \Rightarrow dv = a\,dt \Rightarrow \int_{v_0}^{v} dv = \int_{t_0}^{t} a\,dt$

$\Rightarrow v|_{v_0}^{v} = at|_0^t \Rightarrow v - v_0 = a(t-0) \Rightarrow \boxed{v = at + v_0}$ ★

If we take the initial condition : at $t=0\ s \rightarrow x=x_0$

We have : $\mathrm{v} = \dfrac{dx}{dt} = \underbrace{at + v_0}_{★} \Rightarrow dx = (at + v_0)dt$

$$\Rightarrow \int_{x_0}^{x} dx = \int_{t_0}^{t} (at + v_0)dt \Rightarrow x\Big|_{x_0}^{x} = \frac{at^2}{2} + v_0\, t\Big|_{0}^{t}$$

$$\Rightarrow x - x_0 = \frac{at^2}{2} + v_0 t \Rightarrow \boxed{x = \frac{1}{2}at^2 + v_0 t + x_0}$$

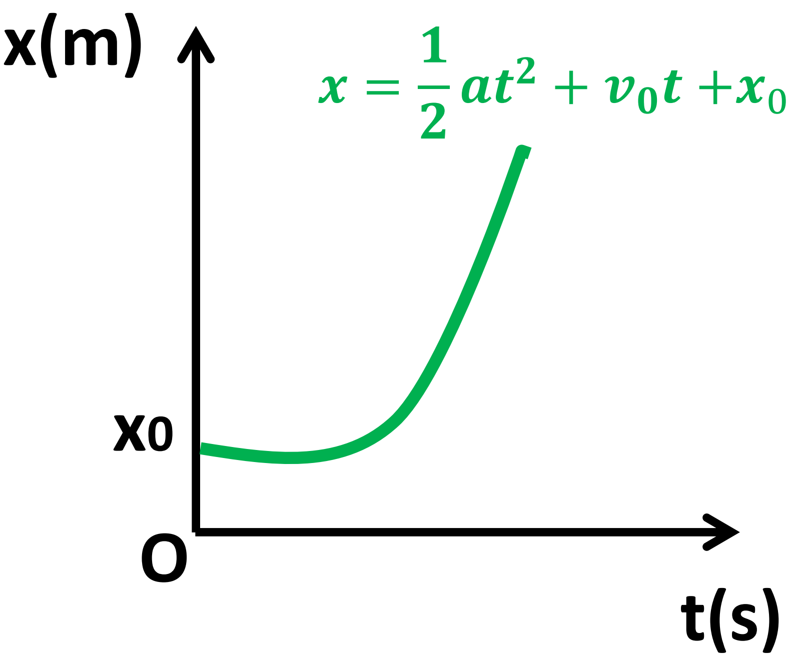
Acceleration (ms^{-2})

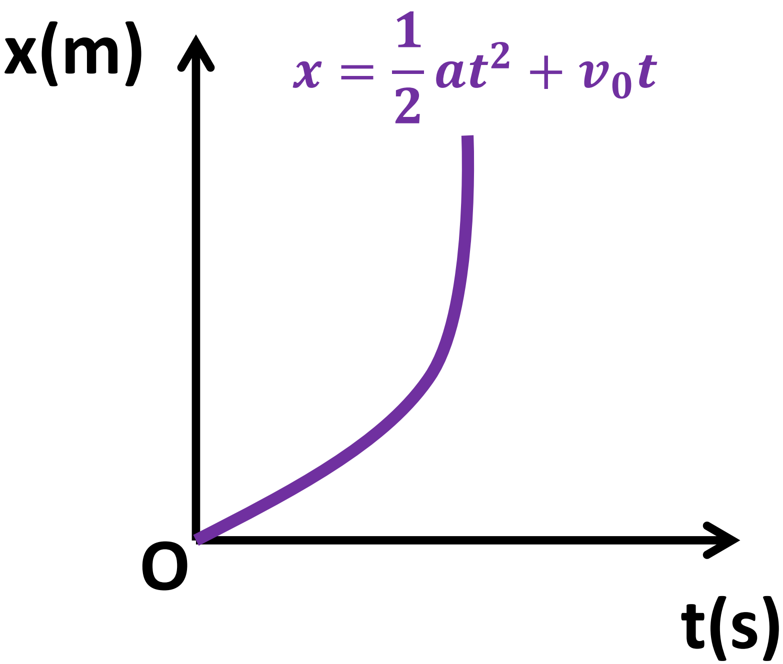
Initial velocity (m/s)

Initial position (m)

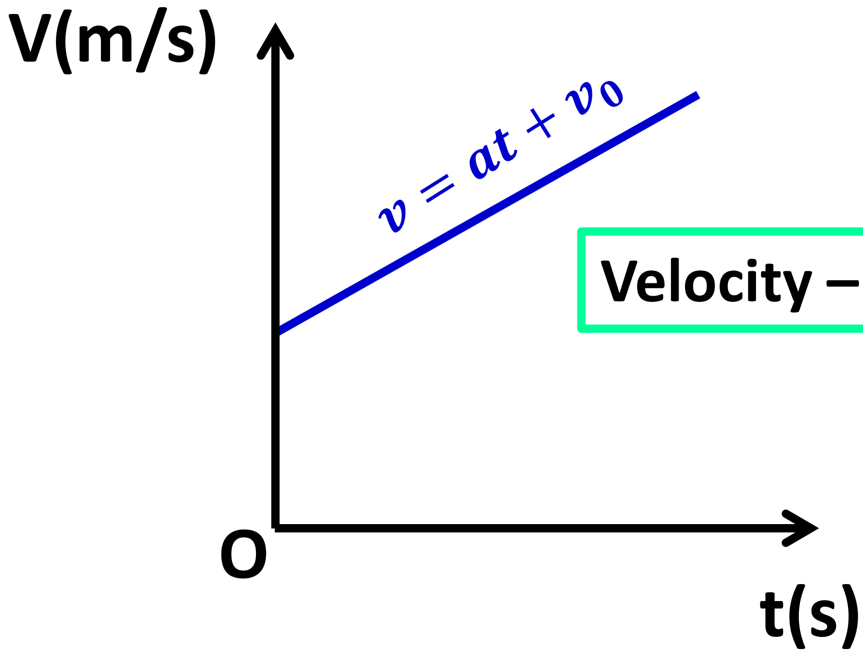
Equation Uniformly Varied Rectilinear motion

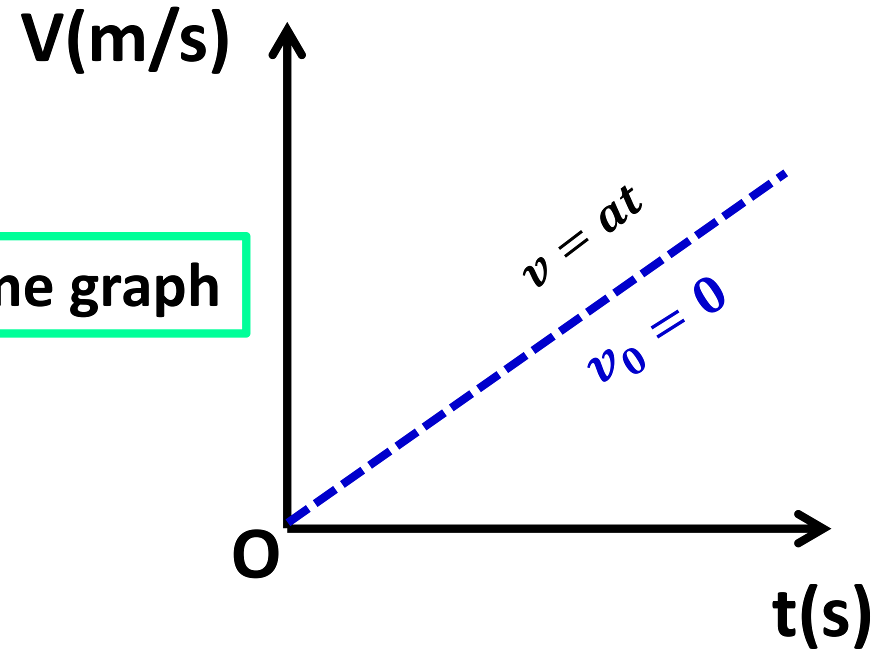
The graphical representation of acceleration, velocity and displacement as a function of time for ***Uniformly Varied Rectilinear Motion*** *as follows.*



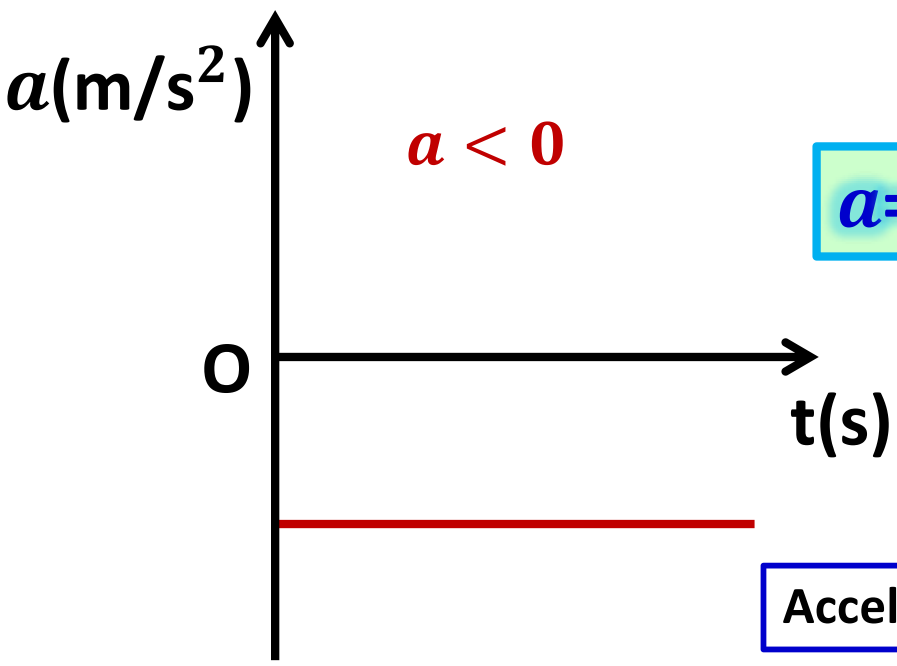


Position – time graph

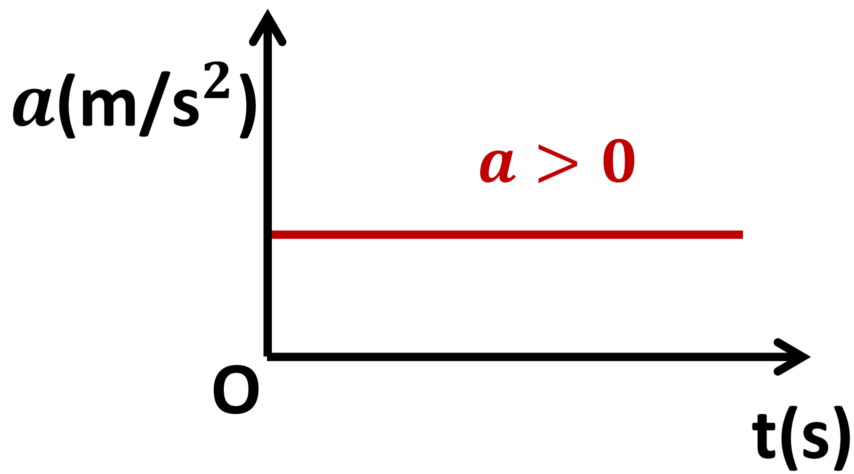




Velocity – time graph



a=C



Acceleration– time graph

Example 2

A point body moves in the axis OX such that its velocity at time 't' is given by :

$$v = 3t - 9$$

1-Find the time equation of this movement knowing that at time t = 0, x =3 m.
2-Deduce the equation of acceleration. What is the nature of the movement?

Solution

1- We have : $v = \frac{dx}{dt} \Rightarrow dx = v\,dt \Rightarrow dx = \underbrace{(3t-9)}_{v}dt$

$$\int_{x_0}^{x} dx = \int_{t_0}^{t} (3t-9)dt \Rightarrow x - x_0 = \frac{3t^2}{2} - 9t \Rightarrow$$

$$x = \frac{3t^2}{2} - 9t + x_0 \quad ☆$$

x_0 ?

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We have : $\begin{cases} t = 0 \\ x = 3m \end{cases}$ (1) → ☆

☆ $\leftrightarrow$ $3m = \dfrac{3(0)^2}{2} - 9(0) + x_0$ $\Longrightarrow$ $x_0 = 3m$ (2)

(2) → ☆ $\Longrightarrow$ $x = \dfrac{3}{2}t^2 - 9t + 3$ **Equation of the movement**

$\dfrac{3}{2}$ → $a(ms^{-2})$

9 → $v_0\ (m/s)$

3 → $x_0(m)$

2-We have : $v = 3t - 9$ $\Longrightarrow$ $a = \dfrac{dv}{dt} = 3(ms^{-2})$ $\Longrightarrow$ $a = 3(ms^{-2})$ =C

The acceleration is constant $a = 3(ms^{-2})$

$\Longrightarrow$ *The movement is Uniformly Varied Rectilinear*

3. Rectilinear motion with variable acceleration

If a material point's trajectory is a straight line and its acceleration is a function of time (a=f(t) variable), then its movement is said to be rectilinear with variable acceleration.

Example 3

A particle is moving on the x-axis. At time t=0, the particle is at the pointe where x=4m.The velocity of the particle at time t is $(-t^2+9t)$ (m/s).

1-Find the equation of acceleration. What is the nature of the movement?

2- Deduce the expression for the displacement of the particle from at time t=0.

Solution

1-We have : $v = -t^2 + 9t \Rightarrow a = \frac{dv}{dt} = -2t + 9 \Rightarrow a = -2t + 9$ a=f(t)

$\Rightarrow$ *The movement is Rectilinear with variable acceleration*

2-We have : $\begin{cases} t = 0 \\ x = 4m \end{cases}$ (1) and $v = -t^2 + 9t$

$v = \dfrac{dx}{dt} \Rightarrow dx = v\,dt \Rightarrow dx = \underbrace{(-t^2 + 9t)}_{v}dt$

$$\int_{x_0}^{x} dx = \int_{t_0}^{t} (-t^2 + 9t)\,dt \Rightarrow x - x_0 = \frac{-t^3}{3} + \frac{9t^2}{2}$$

$$\Rightarrow x = \frac{-t^3}{3} + \frac{9t^2}{2} + x_0 \quad \star$$

$$(1) \rightarrow \star \Rightarrow 4 = \frac{-(0)^3}{3} + \frac{9(0)^2}{2} + x_0 \Rightarrow \boxed{x_0 = 4m} \quad (2)$$

$$(2) \rightarrow \star \Rightarrow \boxed{x = \frac{-t^3}{3} + \frac{9t^2}{2} + 4}$$

4. Sinusoidal rectilinear movement

The motion of a material point is rectilinear sinusoidal if its time equation can be written in the form :

$$x = X_m \sin(\omega t + \varphi) \quad \textit{Or} \quad x = X_m \cos(\omega t + \varphi)$$

x : instantaneous abscissa (m), it varies between two extreme values

$-X_m \leq x \leq +X_m$ *because* $-1 \leq \cos(\omega t + \varphi) \leq +1$

$$\underbrace{-X_m \leq X_m \cos(\omega t + \varphi) \leq X_m}_{x}$$

X_m *:Amplitude (m).*

ω: *Pulse of movement (rad/s).*

φ: *Initial phase (rad).*

$\omega t + \varphi$: *Instant phase (rad).*

Expression of velocity

By deriving the time equation we obtain the expression for instantaneous velocity

We have : $x = X_m \cos(\omega t + \varphi)$ $\Rightarrow$ $v = \dot{x} = \frac{dx}{dt}$

$$[\cos f(x)]' = f'(x)\big(-\sin f(x)\big)$$

$$v = -X_m\,\omega \sin(\omega t + \varphi)$$

This velocity varies between two extreme values :

$$-1 \le \sin(\omega t + \varphi) \le +1 \Rightarrow -X_m\,\omega \le \underbrace{-X_m\,\omega \sin(\omega t + \varphi)}_{v} \le X_m\,\omega$$

$$\Rightarrow -X_m\omega \le v \le +X_m\omega$$

Expression of acceleration

We have :

$$v = -X_m\,\omega \sin(\omega t + \varphi) \Rightarrow a = \ddot{x} = \frac{dv}{dt}$$

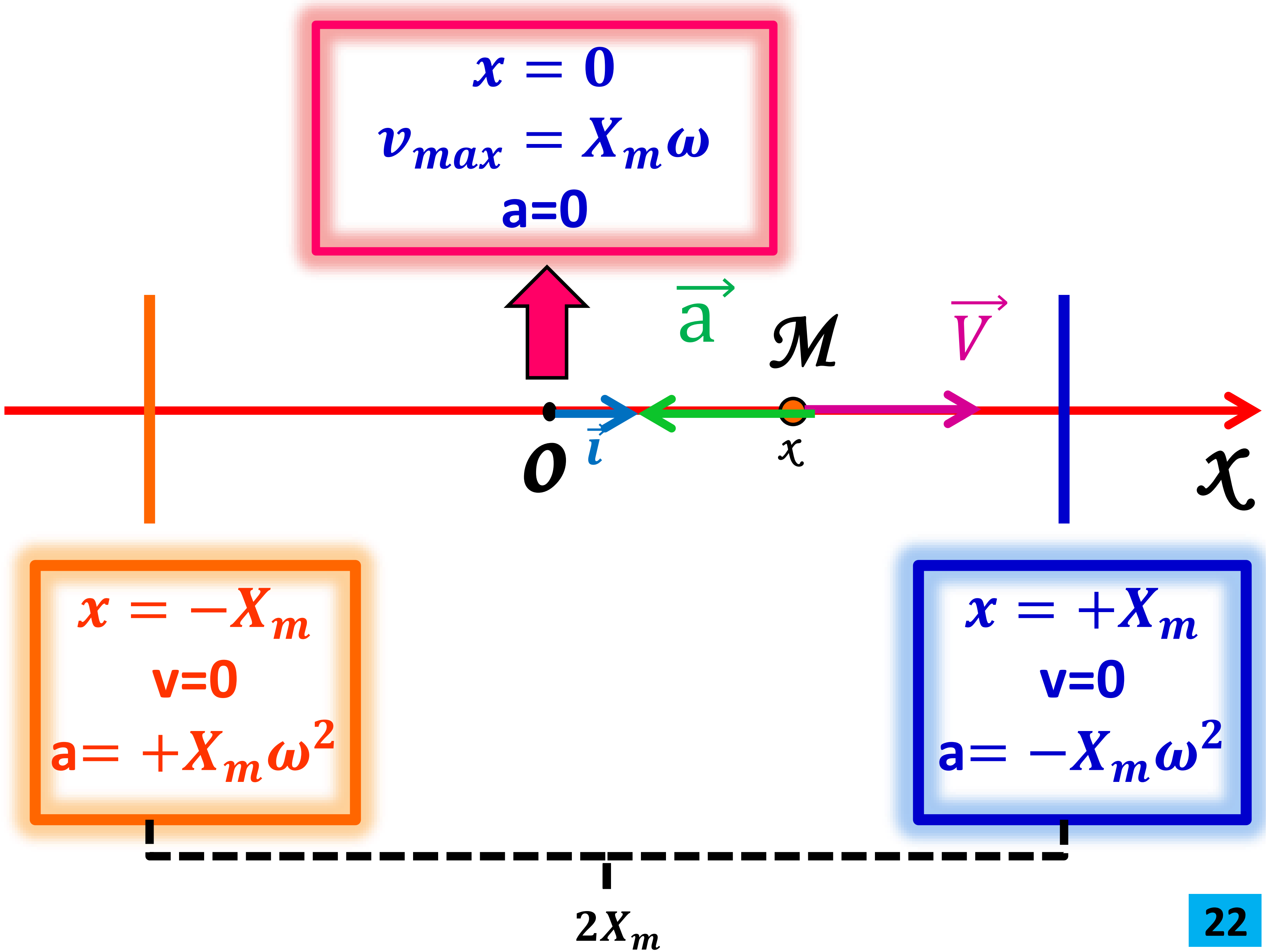
$$\Rightarrow \boxed{a = -X_m\,\omega^2 \cos(\omega t + \varphi)} \Rightarrow \boxed{a = -x\,\omega^2}$$

This acceleration varies between two extreme values :

$$\boxed{-X_m\omega^2 \leq a \leq +X_m\omega^{\,2}}$$

$$-1 \leq \cos(\omega t + \varphi) \leq +1$$

$$-X_m\omega^2 \leq \underbrace{-\omega^2 X_m \cos(\omega t + \varphi)}_{a} \leq X_m\omega^2{}_m$$



$x = 0$
$v_{max} = X_m\omega$
a=0
$\vec{a}$
$\mathcal{M}$
$\vec{V}$
O
$\vec{i}$
x
x
$x = -X_m$
v=0
a= $+X_m\omega^2$
$x = +X_m$
v=0
a= $-X_m\omega^2$
$2X_m$