3^{rd} Tutorial
"Numerical Series and Series of Functions"

2^{nd} Year Engineering (S3)

Exercise 1

Study the convergence of the following numerical series by calculating their sum:

$$\sum_{n=0}^{\infty} \frac{2^{n+1}}{3^n},\ \sum_{n=0}^{\infty} \frac{n+1}{n!}\ \sum_{n=1}^{\infty} \frac{1}{n(n+1)}\ \sum_{n=1}^{\infty} \frac{2n+1}{n^2(n+1)^2}\ \sum_{n=1}^{\infty} \ln \frac{n(n+2)}{(n+1)^2}$$
$$\sum_{n=0}^{\infty} n!,\ \sum_{n=0}^{\infty}(-1)^n\ ,\ \sum_{n=1}^{\infty}\left(\frac{1}{\sqrt{n+1}}-\frac{1}{\sqrt{n}}\right)$$

Exercise 2

Determine the nature of the series $\sum u_n$ with general term u_n using the indicated criterion:

1. Comparison Test: $u_n = \frac{1}{\sqrt{n}}$, $u_n = \sin \frac{1}{n^\alpha}\ \alpha > 0$, $u_n = e^{-\sin \frac{2}{n}}$, $u_n = \ln\left(1+\frac{1}{2^n}\right)$
2. d'Alembert's Test: $u_n = \frac{n^n}{n!}\ u_n = n^3 \sin \frac{\pi}{3^n}\ u_n = \frac{a^n}{n^n},\ a > 0$
3. Cauchy's Root Test: $u_n = \left(\frac{n}{n+1}\right)^{n^2}$, $u_n = 2^{-n}\left(\frac{n+1}{n}\right)^{n^2}$, $u_n = \left(\frac{n+1}{2n-1}\right)^{2n} a^n,\ a > 0$
4. Abel's Criterion: $\sum_{n=1}^{\infty} \frac{(-1)^n}{n} \sin\left(\frac{1}{n}\right)$

Exercise 3

Study the absolute convergence of the following series:

$$\sum_{n=1}^{\infty} \frac{\sin(\alpha n)}{n^2},\ \alpha \in \mathbb{R}\ \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2+\ln n}\ \sum_{n=1}^{\infty} \frac{\cos^2 n}{n}\ \sum_{n=1}^{\infty} \sin\left(n\pi + \frac{1}{n}\right)$$

Exercise 4

Study the pointwise and uniform convergence of :

$$f_n(x) = x^n \text{ on I=[0,1]}\ ,\quad f_n(x) = e^{nx^2}\sin nx + \sqrt{1-x^2} \text{ on I=[-1,1]},\ n \in \mathbb{N}.$$

Exercise 5

Determine the domain of convergence D of each series:

$$\sum_{n\geq 0} \frac{\cos nx}{n!},\ \sum_{n>1} \left|\sin \frac{x^2}{n} - \tan \frac{x^2}{n}\right|^{\frac{1}{2}},\ \sum_{n\geq 1} \frac{n!(x-3)^{2n}}{n^{n+1}}.$$

Dr. A. Sakhri